**Toets voorkennis** **EXTRA: 3 Differentiëren op bladzijde 156 aan het einde van deze uitwerking.**

**a** $f(x) = 2x^4 + x^2 + 5x + 7 \Rightarrow f'(x) = 8x^3 + 2x + 5.$

**b** $g(x) = x^2(2x - 5) = 2x^3 - 5x^2 \Rightarrow g'(x) = 6x^2 - 10x.$

**c** $h(x) = (2x+1)^2 = (2x+1)\cdot(2x+1) = 4x^2 + 2x + 2x + 1 = 4x^2 + 4x + 1 \Rightarrow h'(x) = 8x + 4.$

**d** $s(t) = \frac{1}{6}t^3 + 4t^2 \Rightarrow s'(t) = \frac{1}{2}t^2 + 8t.$

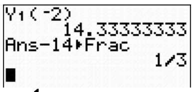
**Toets voorkennis** **EXTRA: 4 Extremen berekenen op bladzijde 157 aan het einde van deze uitwerking.**

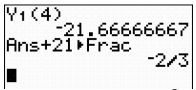
$f(x) = \frac{1}{3}x^3 - x^2 - 8x + 5 \Rightarrow f'(x) = x^2 - 2x - 8.$

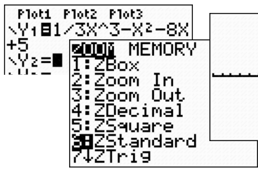
$f'(x) = 0 \Rightarrow x^2 - 2x - 8 = 0$

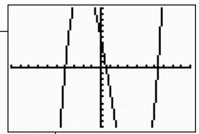
$(x-4)(x+2) = 0$

$x = 4 \vee x = -2.$



Maximum (zie een plot) $f(-2) = 14\frac{1}{3}$ en minimum (zie een plot) $f(4) = -21\frac{2}{3}$.

1a $f(x) = x^2 \Rightarrow f'(x) = 2x$ en $g(x) = 3x - 7 \Rightarrow g'(x) = 3.$
$p(x) = f(x)\cdot g(x) = x^2(3x - 7) = 3x^3 - 7x^2 \Rightarrow p'(x) = 9x^2 - 14x.$

1b $p'(x)$ (zie 1a) $= 9x^2 - 14x$ en $f'(x)\cdot g'(x)$ (zie 1a) $= 2x \cdot 3 = 6x.$ Dus $p'(x) \neq f'(x)\cdot g'(x).$

1c $p'(x) = 9x^2 - 14x$ en $f'(x)\cdot g(x) + f(x)\cdot g'(x)$ (zie 1a) $= 2x\cdot(3x-7) + x^2 \cdot 3 = 6x^2 - 14x + 3x^2 = 9x^2 - 14x.$
Dus $p'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x).$

💻

2a 💻 $f(x) = x^2(2x-1) \Rightarrow f'(x) = 2x(2x-1) + x^2 \cdot 2 = 2x(2x-1) + 2x^2.$

2b 💻 $g(x) = 2x^3(x^2-3) \Rightarrow g'(x) = 6x^2(x^2-3) + 2x^3 \cdot 2x = 6x^2(x^2-3) + 4x^4.$

2c 💻 $h(x) = (x^2-1)(x^2+3) \Rightarrow h'(x) = 2x(x^2+3) + (x^2-1)\cdot 2x.$

2d 💻 $j(x) = (2x^3+1)(3x^2-1) \Rightarrow j'(x) = 6x^2(3x^2-1) + (2x^3+1)\cdot 6x.$

3a $f(x) = (2-3x^2)(2+7x) \Rightarrow f'(x) = -6x(2+7x) + (2-3x^2)\cdot 7.$

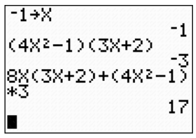
3b $g(x) = (2x-5)^2 = (2x-5)(2x-5) \Rightarrow g'(x) = 2(2x-5) + (2x-5)\cdot 2 = 4(2x-5).$

3c $h(x) = (x^2-3x)(x^3+x^2+x) \Rightarrow h'(x) = (2x-3)(x^3+x^2+x) + (x^2-3x)(3x^2+2x+1).$

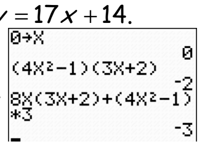
3d $j(x) = (3x^2-4)^2 = (3x^2-4)(3x^2-4) \Rightarrow j'(x) = 6x(3x^2-4) + (3x^2-4)\cdot 6x = 12x(3x^2-4).$

4 $g(x) = c\cdot f(x) \Rightarrow g'(x) = [c]'\cdot f(x) + c\cdot[f(x)]' = 0\cdot f(x) + c\cdot f'(x) = c\cdot f'(x).$

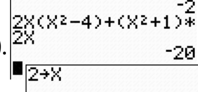
5a $f(x) = (4x^2-1)(3x+2) \Rightarrow f'(x) = 8x(3x+2) + (4x^2-1)\cdot 3.$
$y_A = f(-1) = -3$ en $rc_{raaklijn} = f'(-1) = 17.$
$k$: $y = 17x + b$ door $A(-1,\,-3) \Rightarrow 17\cdot -1 + b = -3 \Rightarrow b = -3 + 17 = 14.$ Dus $k$: $y = 17x + 14.$

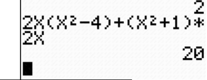



5b $B$ op de $y$-as $(x = 0) \Rightarrow y_B = f(0) = -2$ en $rc_{raaklijn} = f'(0) = -3.$
$l$: $y = -3x + b$ door $B(0,\,-2) \Rightarrow -3\cdot 0 + b = -2 \Rightarrow b = -2.$ Dus $l$: $y = -3x - 2.$



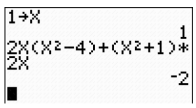

6a $f(x) = (x^2+1)(x^2-4) = 0 \Rightarrow x^2 = -1$ (kan niet) $\vee\ x^2 = 4 \Rightarrow x = -2 \vee x = 2.$ Dus $A(-2,\,0)$ en $B(2,\,0).$
$f(x) = (x^2+1)(x^2-4) \Rightarrow f'(x) = 2x(x^2-4) + (x^2+1)\cdot 2x.$ Dus $f'(-2) = -20$ en $f'(2) = 20.$
$k$: $y = -20x + b$ door $A(-2,\,0) \Rightarrow -20\cdot -2 + b = 0 \Rightarrow b = -40.$ Dus $k$: $y = -20x - 40.$
$l$: $y = 20x + b$ door $B(2,\,0) \Rightarrow 20\cdot 2 + b = 0 \Rightarrow b = -40.$ Dus $l$: $y = 20x - 40.$

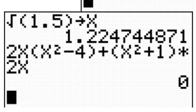



6b $f'(1) = -2 \neq 0 \Rightarrow$ de grafiek van $f$ heeft geen extreme waarde voor $x = 1.$

6c $f'(\sqrt{1\frac{1}{2}}) = 0 \Rightarrow$ de grafiek van $f$ heeft een extreme waarde voor $x = \sqrt{1\frac{1}{2}}.$



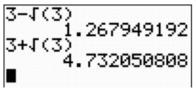

7a $A = O(PQRS) = PQ\cdot PS.$ De grafiek van $f$ snijdt de $x$-as in $O(0,\,0)$ en $(6,\,0)$ $(-x^2 + 6x = -x(x-6) = 0 \Rightarrow x = 0 \vee x = 6).$
$PQ = 6 - 2\cdot OP = 6 - 2\cdot x_P = 6 - 2p$ en $PS = y_S = -p^2 + 6p.$ Dus $A = PQ\cdot PS = (6-2p)(-p^2+6p).$

7b $A = (6-2p)(-p^2+6p) \Rightarrow A'(p) = \frac{dA}{dp} = -2(-p^2+6p) + (6-2p)(-2p+6)$
$= 2p^2 - 12p - 12p + 36 + 4p^2 - 12p = 6p^2 - 36p + 36.$

**7c** $\frac{dA}{dp} = 0 \Rightarrow 6p^2 - 36p + 36 = 0 \Rightarrow p^2 - 6p + 6 = 0$ met $D = (-6)^2 - 4 \cdot 1 \cdot 6 = 36 - 24 = 12$

$p = \frac{6-\sqrt{12}}{2 \cdot 1} = \frac{6 - 2\sqrt{3}}{2} = 3 - \sqrt{3} \vee p = \frac{6+\sqrt{12}}{2} = 3 + \sqrt{3}$ (> 3 voldoet niet).

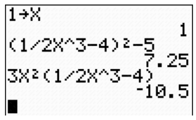
We zoeken dus $p = 3 - \sqrt{3}$. (de enige kandidaat voor het maximum)




**8a** $f(x) = (\frac{1}{2}x^3 - 4)^2 - 5 = (\frac{1}{2}x^3 - 4)(\frac{1}{2}x^3 - 4) - 5 \Rightarrow f'(x) = \frac{3}{2}x^2(\frac{1}{2}x^3 - 4) + (\frac{1}{2}x^3 - 4) \cdot \frac{3}{2}x^2 = 3x^2(\frac{1}{2}x^3 - 4)$.

$y_A = f(1) = 7{,}25$ en rc $= f'(1) = -10{,}5$.

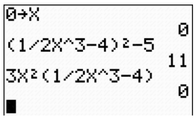
$k$: $y = -10{,}5x + b$ door $A(1;\ 7{,}25) \Rightarrow -10{,}5 \cdot 1 + b = 7{,}25 \Rightarrow b = 17{,}75$. Dus $k$: $y = -10{,}5x + 17{,}75$.




**8b** $y_B = f(0) = 11$ en rc $= f'(0) = 0$.

$k$: $y = 0x + b = b$ door $B(0, 11)$.

Dus $k$: $y = 11$.

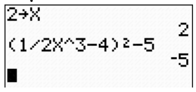



**8c** $f'(x) = 0 \Rightarrow 3x^2(\frac{1}{2}x^3 - 4) = 0$

$x = 0$ (hoort niet bij de top) $\vee$ $\frac{1}{2}x^3 = 4$

$x^3 = 8$

$x = 2$ met $f(2) = -5 \Rightarrow T(2,\ -5)$.



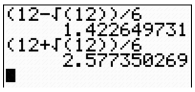

**9a** $O(\triangle ABC) = \frac{1}{2} \cdot AC \cdot AB = \frac{1}{2} \cdot (OC - OA) \cdot f(p) = \frac{1}{2}(4 - p)(p^2 - 2p + 3) = (2 - \frac{1}{2}p)(p^2 - 2p + 3)$.

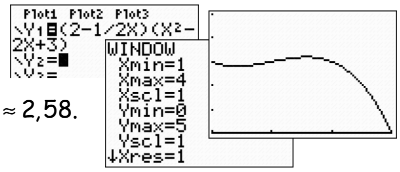
**9b** $O'(p) = \frac{dO}{dp} = -\frac{1}{2}(p^2 - 2p + 3) + (2 - \frac{1}{2}p)(2p - 2) = -\frac{1}{2}p^2 + p - \frac{3}{2} + 4p - 4 - p^2 + p = -1\frac{1}{2}p^2 + 6p - 5\frac{1}{2}$.

$O'(p) = 0 \Rightarrow -1\frac{1}{2}p^2 + 6p - 5\frac{1}{2} = 0$ (keer $-2$)

$3p^2 - 12p + 11 = 0$ met $D = (-12)^2 - 4 \cdot 3 \cdot 11 = 144 - 132 = 12$

$p = \frac{12 - \sqrt{12}}{2 \cdot 3}$ (hoort bij een minimum van $O$) $\vee$ $p = \frac{12+\sqrt{12}}{6} \approx 2{,}58$. Dus we zoeken $p \approx 2{,}58$.




**10a** $\frac{1}{x^3} = x^{-3}$. $\frac{5}{x^4} = 5 \cdot \frac{1}{x^4} = 5x^{-4}$. $\frac{1}{3x^2} = \frac{1}{3} \cdot \frac{1}{x^2} = \frac{1}{3} \cdot x^{-2}$.

**10b** $x^{-4} = \frac{1}{x^4}$. $3x^{-2} = 3 \cdot \frac{1}{x^2} = \frac{3}{x^2}$. $\frac{1}{7} \cdot x^{-6} = \frac{1}{7} \cdot \frac{1}{x^6} = \frac{1}{7x^6}$.

**11a** $h(x) = x^2 \cdot x^{-2} \Rightarrow h'(x) = 2x \cdot x^{-2} + x^2 \cdot [x^{-2}]'$.

**11b** $h(x) = x^2 \cdot x^{-2} = x^{2+-2} = x^0 = 1 \Rightarrow h'(x) = 0$.

**11c** $h'(x) = 0$ (zie 1b en gebruik 1a)

$2x \cdot x^{-2} + x^2 \cdot [x^{-2}]' = 0$

$2x^{-1} + x^2 \cdot [x^{-2}]' = 0$

$x^2 \cdot [x^{-2}]' = -2x^{-1}$

dus $[x^{-2}]' = \frac{-2x^{-1}}{x^2}$.

**11d** $[x^{-2}]' = \frac{-2x^{-1}}{x^2} = -2x^{-1-2} = -2x^{-3}$.

Dus $[x^n]' = \ldots\ldots\ldots\ldots = nx^{n-1}$ geldt ook voor $n = -2$.

**12a** $f(x) = \frac{5}{x^3} = 5 \cdot \frac{1}{x^3} = 5x^{-3} \Rightarrow f'(x) = -15x^{-4} = -15 \cdot \frac{1}{x^4} = -\frac{15}{x^4}$.

**12b** $g(x) = \frac{1}{5x^3} = \frac{1}{5} \cdot \frac{1}{x^3} = \frac{1}{5}x^{-3} \Rightarrow g'(x) = -\frac{3}{5}x^{-4} = -\frac{3}{5} \cdot \frac{1}{x^4} = -\frac{3}{5x^4}$.

**12c** $h(x) = 5x^2 - \frac{5}{x^2} = 5x^2 - 5 \cdot \frac{1}{x^2} = 5x^2 - 5x^{-2} \Rightarrow h'(x) = 10x + 10x^{-3} = 10x + 10 \cdot \frac{1}{x^3} = 10x + \frac{10}{x^3}$.

**13a** $f(x) = \frac{x^4 - 5}{x^3} = \frac{x^4}{x^3} - \frac{5}{x^3} = x - 5x^{-3} \Rightarrow f'(x) = 1 + 15x^{-4} = 1 + \frac{15}{x^4}$.

**13b** $g(x) = \frac{2x^2 - 3}{x^3} = \frac{2x^2}{x^3} - \frac{3}{x^3} = 2x^{-1} - 3x^{-3} \Rightarrow g'(x) = -2x^{-2} + 9x^{-4} = -\frac{2}{x^2} + \frac{9}{x^4}$.

**13c** $h(x) = \frac{x+2}{3x} = \frac{x}{3x} + \frac{2}{3x} = \frac{1}{3} + \frac{2}{3}x^{-1} \Rightarrow h'(x) = 0 - \frac{2}{3}x^{-2} = -\frac{2}{3x^2}$.

**14a** $f(x) = \frac{2x - 1}{3x^2} = \frac{2x}{3x^2} - \frac{1}{3x^2} = \frac{2}{3}x^{-1} - \frac{1}{3}x^{-2} \Rightarrow f'(x) = -\frac{2}{3}x^{-2} + \frac{2}{3}x^{-3} = -\frac{2}{3x^2} + \frac{2}{3x^3}$.

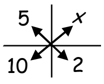
**14b** $g(x) = 6 - \frac{x^2 - 1}{x} = 6 - \frac{x^2}{x} + \frac{1}{x} = 6 - x + x^{-1} \Rightarrow g'(x) = 0 - 1 - x^{-2} = -1 - \frac{1}{x^2}$.

**14c** $h(x) = \frac{5}{2x^2} - \frac{2x^2}{5} = \frac{5}{2}x^{-2} - \frac{2}{5}x^2 \Rightarrow h'(x) = -5x^{-3} - \frac{4}{5}x = -\frac{5}{x^3} - \frac{4}{5}x$.

**15a** $5 \cdot 2 = 10$ en $10 \cdot 1 = 10$, dus $5 \cdot 2 = 10 \cdot 1$.

**15b** $\frac{5}{10} = \frac{x}{2}$ (kruisproducten nemen)

$5 \cdot 2 = 10 \cdot x$.




**15c** $\frac{5}{6} = \frac{3}{x}$ (kruiselings vermenigvuldigen)

$5x = 18$

$x = \frac{18}{5} = 3\frac{3}{5}$.

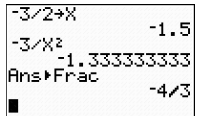
16a $\frac{4}{x^2} = \frac{1}{9}$ (kruisproducten)
$x^2 = 36$
$x = -6 \vee x = 6.$

16b $\frac{x-3}{x} = \frac{2}{7}$ (kruisproducten)
$7(x-3) = 2x$
$7x - 21 = 2x$
$5x = 21$
$x = \frac{21}{5} = 4\frac{1}{5}.$

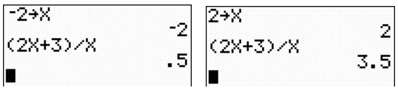
16c $\frac{x}{x-4} = \frac{3}{8}$ (kruisproducten)
$3(x-4) = 8x$
$3x - 12 = 8x$
$-5x = 12 \Rightarrow x = \frac{12}{-5} = -2\frac{2}{5}.$

16d $3 - \frac{6}{x^2} = 1\frac{1}{2}$
$1\frac{1}{2} = \frac{3}{2} = \frac{6}{x^2}$ (kruisproducten)
$3x^2 = 12$
$x^2 = 4 \Rightarrow x = -2 \vee x = 2.$

17a $f(x) = \frac{2x+3}{x} = \frac{2x}{x} + \frac{3}{x} = 2 + 3x^{-1} \Rightarrow f'(x) = -3x^{-2} = -\frac{3}{x^2}.$
Snijden met de $x$-as $(y=0) \Rightarrow f(x) = 0 \Rightarrow \frac{2x+3}{x} = \frac{0}{1} \Rightarrow 2x + 3 = 0 \Rightarrow 2x = -3.$ Dus $x_A = -\frac{3}{2}.$
$y_A = f(-\frac{3}{2}) = 0$ (y = 0 was gegeven, zie de regel hierboven) en $rc_{\text{raaklijn}} = f'(-\frac{3}{2}) = -\frac{4}{3}.$
$k$: $y = -\frac{4}{3}x + b$ door $A(-\frac{3}{2}, 0) \Rightarrow -\frac{4}{3} \cdot -\frac{3}{2} + b = 0 \Rightarrow b = 0 - \frac{4}{3} \cdot \frac{3}{2} = -\frac{12}{6} = -2.$ Dus $k$: $y = -\frac{4}{3}x - 2.$




17b $rc_{\text{raaklijn}} = f'(x) = -\frac{3}{4} \Rightarrow -\frac{3}{x^2} = -\frac{3}{4} \Rightarrow \frac{3}{x^2} = \frac{3}{4} \Rightarrow 3x^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = -2 \vee x = 2.$
$y_B = f(-2) = \frac{1}{2}$ en $y_C = f(2) = 3\frac{1}{2}.$
De raakpunten zijn $B(-2, \frac{1}{2})$ en $C(2, 3\frac{1}{2}).$

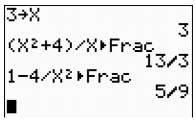



18a $\frac{x+1}{x-1} = \frac{x+3}{x}$ (kruisproducten)
$x(x+1) = (x+3)(x-1)$
$x^2 + x = x^2 + 2x - 3$
$-x = -3$
$x = 3.$

18b $\frac{x}{x+2} = \frac{3}{x-2}$ (kruisproducten)
$x(x-2) = 3(x+2)$
$x^2 - 2x = 3x + 6$
$x^2 - 5x - 6 = 0$
$(x-6)(x+1) = 0$
$x = 6 \vee x = -1.$

18c $\frac{x}{x+4} = \frac{1}{2x-1}$ (kruisproducten)
$x(2x-1) = x + 4$
$2x^2 - x = x + 4$
$2x^2 - 2x - 4 = 0$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2 \vee x = -1.$

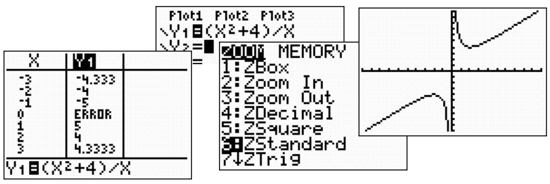
18d $\frac{4x}{x+2} + 3 = x$
$\frac{4x}{x+2} = \frac{x-3}{1}$ (kruisproducten)
$(x-3)(x+2) = 4x$
$x^2 - 3x + 2x - 6 = 4x$
$x^2 - 5x - 6 = 0$
$(x-6)(x+1) = 0$
$x = 6 \vee x = -1.$




19a $f(x) = \frac{x^2+4}{x} = \frac{x^2}{x} + \frac{4}{x} = x + 4x^{-1} \Rightarrow f'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2}.$
$y_A = f(3) = \frac{13}{3}$ en $rc_{\text{raaklijn}} = f'(3) = \frac{5}{9}.$
$k$: $y = \frac{5}{9}x + b$ door $A(3, \frac{13}{3}) \Rightarrow \frac{5}{9} \cdot 3 + b = \frac{13}{3} \Rightarrow b = \frac{13}{3} - \frac{5}{9} \cdot 3 = \frac{13}{3} - \frac{5}{3} = \frac{8}{3}.$ Dus $k$: $y = \frac{5}{9}x + \frac{8}{3}.$

19b $rc_{\text{raaklijn}} = f'(x) = -3 \Rightarrow 1 - \frac{4}{x^2} = -3 \Rightarrow -\frac{4}{x^2} = \frac{-4}{1} \Rightarrow -4x^2 = -4 \Rightarrow x^2 = 1 \Rightarrow x = -1 \vee x = 1.$
$y_B = f(-1) = -5$ en $y_C = f(1) = 5.$
De raakpunten zijn $B(-1, -5)$ en $C(1, 5).$

19c $f'(x) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0 \Rightarrow \frac{1}{1} = \frac{4}{x^2} \Rightarrow x^2 = 4 \Rightarrow x = -2 \vee x = 2.$
Maximum (zie een plot) $f(-2) = -4$ en minimum (zie een plot) $f(2) = 4.$




19d $rc_{\text{raaklijn}} = f'(x) = 2 \Rightarrow 1 - \frac{4}{x^2} = 2 \Rightarrow -\frac{4}{x^2} = \frac{1}{1} \Rightarrow x^2 = -4$ (kan niet) $\Rightarrow$ geen oplossing.

**Toets voorkennis**

**EXTRA: 5 Machten met positieve en/of gebroken exponenten op bladzijden 158 en 159 aan het einde van deze uitwerking.**

**1a** $x^{-3} = \frac{1}{x^3}.$

**1b** $x^{\frac{5}{6}} = \sqrt[6]{x^5}.$

**1c** $x^{1\frac{1}{3}} = x^1 \cdot x^{\frac{1}{3}} = x \cdot \sqrt[3]{x}.$

**1d** $x^{-1\frac{1}{2}} = \frac{1}{x^{1\frac{1}{2}}} = \frac{1}{x^1 \cdot x^{\frac{1}{2}}} = \frac{1}{x \cdot \sqrt{x}}.$

**2a** $x^2 \cdot \sqrt{x} = x^2 \cdot x^{\frac{1}{2}} = x^{2\frac{1}{2}}$.

**2b** $\frac{x^{-1}}{x^2} = x^{-1-2} = x^{-3}$.

**2c** $\frac{1}{x^3 \cdot \sqrt{x}} = \frac{1}{x^3 \cdot x^{\frac{1}{2}}} = \frac{1}{x^{3\frac{1}{2}}} = x^{-3\frac{1}{2}}$.

**2d** $\frac{1}{x \cdot \sqrt[3]{x}} = \frac{1}{x^1 \cdot x^{\frac{1}{3}}} = \frac{1}{x^{1\frac{1}{3}}} = x^{-1\frac{1}{3}}$.

20a $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$ (links en rechts de afgeleide nemen)
$x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' + [x^{\frac{1}{2}}]' \cdot x^{\frac{1}{2}} = 1$
$2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$

20b $2 \cdot x^{\frac{1}{2}} \cdot [x^{\frac{1}{2}}]' = 1$
$[x^{\frac{1}{2}}]' = \frac{1}{2 \cdot x^{\frac{1}{2}}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2} \cdot x^{-\frac{1}{2}}$.

Uit het hoofd leren:
$[\sqrt{x}]' = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{x^{\frac{1}{2}}} = \frac{1}{2 \cdot \sqrt{x}}$

21a $f(x) = x + \sqrt{x} = x + x^{\frac{1}{2}} \Rightarrow f'(x) = 1 + \frac{1}{2} x^{-\frac{1}{2}} = 1 + \frac{1}{2x^{\frac{1}{2}}} = 1 + \frac{1}{2 \cdot \sqrt{x}}$.

21b $g(x) = x \cdot \sqrt[3]{x} = x \cdot x^{\frac{1}{3}} = x^{1\frac{1}{3}} \Rightarrow g'(x) = 1\frac{1}{3} x^{\frac{1}{3}} = 1\frac{1}{3} \cdot \sqrt[3]{x}$.

21c $h(x) = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}} \Rightarrow h'(x) = -\frac{1}{2} x^{-1\frac{1}{2}} = -\frac{1}{2x^{1\frac{1}{2}}} = -\frac{1}{2x \cdot x^{\frac{1}{2}}} = -\frac{1}{2x \cdot \sqrt{x}}$.

21d $j(x) = x^3 \cdot \sqrt[5]{x^3} = x^3 \cdot x^{\frac{3}{5}} = x^{3\frac{3}{5}} \Rightarrow j'(x) = 3\frac{3}{5} x^{2\frac{3}{5}} = 3\frac{3}{5} x^2 \cdot x^{\frac{3}{5}} = 3\frac{3}{5} x^2 \cdot \sqrt[5]{x^3}$.

21e $k(x) = x^2 \cdot \sqrt[4]{x} = x^2 \cdot x^{\frac{1}{4}} = x^{2\frac{1}{4}} \Rightarrow k'(x) = 2\frac{1}{4} x^{1\frac{1}{4}} = 2\frac{1}{4} x \cdot x^{\frac{1}{4}} = 2\frac{1}{4} x \cdot \sqrt[4]{x}$.

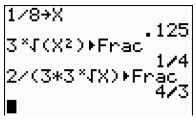
21f $l(x) = (x^2 + 1) \cdot (1 + \sqrt{x})$ (productregel) $\Rightarrow g'(x) = 2x \cdot (1 + \sqrt{x}) + (x^2 + 1) \cdot \frac{1}{2 \cdot \sqrt{x}} = 2x(1 + \sqrt{x}) + \frac{x^2 + 1}{2 \cdot \sqrt{x}}$.

22a $f(x) = \frac{4x^2+1}{x \cdot \sqrt{x}} = \frac{4x^2+1}{x \cdot x^{\frac{1}{2}}} = \frac{4x^2+1}{x^{1\frac{1}{2}}} = \frac{4x^2}{x^{1\frac{1}{2}}} + \frac{1}{x^{1\frac{1}{2}}} = 4x^{\frac{1}{2}} + x^{-1\frac{1}{2}} \Rightarrow f'(x) = 2x^{-\frac{1}{2}} - 1\frac{1}{2} x^{-2\frac{1}{2}} = \frac{2}{x^{\frac{1}{2}}} - \frac{3}{2x^2 \cdot x^{\frac{1}{2}}} = \frac{2}{\sqrt{x}} - \frac{3}{2x^2 \cdot \sqrt{x}}$.

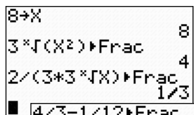
22b $g(x) = \frac{x-4}{\sqrt[3]{x}} = \frac{x-4}{x^{\frac{1}{3}}} = \frac{x}{x^{\frac{1}{3}}} - \frac{4}{x^{\frac{1}{3}}} = x^{\frac{2}{3}} - 4x^{-\frac{1}{3}} \Rightarrow g'(x) = \frac{2}{3} x^{-\frac{1}{3}} + \frac{4}{3} x^{-1\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}} + \frac{4}{3x \cdot x^{\frac{1}{3}}} = \frac{2}{3 \cdot \sqrt[3]{x}} + \frac{4}{3x \cdot \sqrt[3]{x}}$.

22c $h(x) = (x\sqrt{x} - 3)^2 = (x^{1\frac{1}{2}} - 3)^2 = (x^{1\frac{1}{2}} - 3)(x^{1\frac{1}{2}} - 3) \Rightarrow h'(x) = 1\frac{1}{2} x^{\frac{1}{2}} (x^{1\frac{1}{2}} - 3) + (x^{1\frac{1}{2}} - 3) \cdot 1\frac{1}{2} x^{\frac{1}{2}} = 3\sqrt{x}(x\sqrt{x} - 3)$.

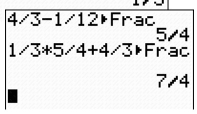
23 $f(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}} \Rightarrow f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3x^{\frac{1}{3}}} = \frac{2}{3 \cdot \sqrt[3]{x}}$.




$y_A = f(\frac{1}{8}) = \frac{1}{4}$ en $rc_{raaklijn} = f'(\frac{1}{8}) = \frac{4}{3}$.
$k$: $y = \frac{4}{3}x + b$ door $A(\frac{1}{8}, \frac{1}{4}) \Rightarrow \frac{4}{3} \cdot \frac{1}{8} + b = \frac{1}{4} \Rightarrow b = \frac{1}{4} - \frac{4}{3} \cdot \frac{1}{8} = \frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$. Dus $k$: $y = \frac{4}{3}x + \frac{1}{12}$.
$y_B = f(8) = 4$ en $rc_{raaklijn} = f'(8) = \frac{1}{3}$.
$l$: $y = \frac{1}{3}x + b$ door $B(8, 4) \Rightarrow \frac{1}{3} \cdot 8 + b = 4 \Rightarrow b = 4 - \frac{8}{3} = \frac{12}{3} - \frac{8}{3} = \frac{4}{3}$. Dus $l$: $y = \frac{1}{3}x + \frac{4}{3}$.




$k$ snijden met $l$ geeft $\frac{4}{3}x + \frac{1}{12} = \frac{1}{3}x + \frac{4}{3}$
$x = \frac{4}{3} - \frac{1}{12} = \frac{16}{12} - \frac{1}{12} = \frac{15}{12} = \frac{5}{4}$ met $y = \frac{1}{3} \cdot \frac{5}{4} + \frac{4}{3} = \frac{5}{12} + \frac{16}{12} = \frac{21}{12} = \frac{7}{4}$. Dus $C(\frac{5}{4}, \frac{7}{4})$.




24a $f(x) = x\sqrt{x} - 3x = x \cdot x^{\frac{1}{2}} - 3x = x^{1\frac{1}{2}} - 3x \Rightarrow f'(x) = 1\frac{1}{2} x^{\frac{1}{2}} - 3 = 1\frac{1}{2}\sqrt{x} - 3$.
$f'(x) = 0 \Rightarrow 1\frac{1}{2}\sqrt{x} - 3 = 0 \Rightarrow 1\frac{1}{2}\sqrt{x} = 3 \Rightarrow \sqrt{x} = 2$ (kwadrateren) $\Rightarrow x = 4$ (de enige kandidaat voor een minimum).
Het minimum is $f(4) = 4 \cdot \sqrt{4} - 3 \cdot 4 = 4 \cdot 2 - 3 \cdot 4 = -4$.

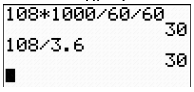
24b $rc_{raaklijn} = f'(0) = -3$ geeft $k$: $y = -3x + b$ door $O(0, 0) \Rightarrow -3 \cdot 0 + b = 0 \Rightarrow b = 0$. Dus $k$: $y = -3x$.

24c $f'(x) = 3 \Rightarrow 1\frac{1}{2}\sqrt{x} - 3 = 3 \Rightarrow 1\frac{1}{2}\sqrt{x} = 6 \Rightarrow \sqrt{x} = 4$ (kwadrateren) $\Rightarrow x = 16 = x_A$.
$y_A = f(16) = 16 \cdot \sqrt{16} - 3 \cdot 16 = 16 \cdot 4 - 3 \cdot 16 = 16$. Dus $A(16, 16)$.
$l$: $y = 3x + b$ door $A(16, 16) \Rightarrow 3 \cdot 16 + b = 16 \Rightarrow b = 16 - 3 \cdot 16 = -32$. Dus $l$: $y = 3x - 32$.

25a $s(t) = 10t \cdot \sqrt{t} = 10t \cdot t^{\frac{1}{2}} = 10t^{1\frac{1}{2}} \Rightarrow \frac{ds}{dt} = s'(t) = 15t^{\frac{1}{2}} = 15 \cdot \sqrt{t}$.
Dus $\left[\frac{ds}{dt}\right]_{t=1} = 15 \cdot \sqrt{1} = 15 \cdot 1 = 15$.

25b De snelheid is $\left[\frac{ds}{dt}\right]_{t=8} = 15 \cdot \sqrt{8}$ m/s.

25c $\quad$ 108 km/u is $\frac{108\cdot 1000}{60\cdot 60}=30$ m/s.




$\frac{ds}{dt}=30$

$15\cdot\sqrt{t}=30$

$\sqrt{t}=2\Rightarrow t=4$. Dus na 4 seconden.

25d $\quad$ De formule $s(t)=10t\cdot\sqrt{t}$ geldt voor $0\le t\le 9$.

Na 9 seconden is $s(9)=10\cdot 9\cdot\sqrt{9}=90\cdot 3=270$ meter afgelegd.

De snelheid vanaf $t=9$ is $s'(9)=15\cdot\sqrt{9}=15\cdot 3=45$ m/s.

Van $t=9$ tot $t=60$ legt de trein $(60-9)\cdot 45=2295$ meter af.

In de eerste minuut legt de trein $270+2295=2565$ meter af.

26a $\quad f(x)=\frac{x^3+2}{\sqrt{x}}=\frac{x^3+2}{x^{\frac{1}{2}}}=\frac{x^3}{x^{\frac{1}{2}}}+\frac{2}{x^{\frac{1}{2}}}=x^{2\frac{1}{2}}+2x^{-\frac{1}{2}}\Rightarrow f'(x)=2\frac{1}{2}x^{1\frac{1}{2}}-x^{-1\frac{1}{2}}=2\frac{1}{2}x\cdot\sqrt{x}-\frac{1}{x^{1\frac{1}{2}}}=2\frac{1}{2}x\cdot\sqrt{x}-\frac{1}{x\cdot\sqrt{x}}$.

$y_A=f(1)=\frac{3}{1}=3$ en $rc_{raaklijn}=f'(1)=2\frac{1}{2}-\frac{1}{1}=2\frac{1}{2}-1=1\frac{1}{2}$.

$k$: $y=1\frac{1}{2}x+b$ door $A(1,3)\Rightarrow 1\frac{1}{2}\cdot 1+b=3\Rightarrow b=3-1\frac{1}{2}=1\frac{1}{2}$. Dus $k$: $y=1\frac{1}{2}x+1\frac{1}{2}$.

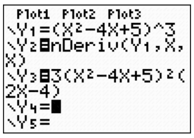
26b $\quad f'(x)=0\Rightarrow 2\frac{1}{2}x\cdot\sqrt{x}-\frac{1}{x\cdot\sqrt{x}}=0\Rightarrow\frac{2\frac{1}{2}x\cdot\sqrt{x}}{1}=\frac{1}{x\cdot\sqrt{x}}\Rightarrow 2\frac{1}{2}x^2\cdot x=1\Rightarrow\frac{5}{2}x^3=1\Rightarrow x^3=\frac{2}{5}\Rightarrow x=\sqrt[3]{\frac{2}{5}}$. Dus $p=\frac{2}{5}$.

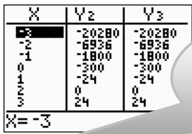
26c $\quad f\left(\sqrt[3]{\frac{2}{5}}\right)=\frac{\frac{2}{5}+2}{\sqrt{\sqrt[3]{\frac{2}{5}}}}=\frac{2\frac{2}{5}}{\sqrt[6]{\frac{2}{5}}}$. Dus $a=2\frac{2}{5}$, $b=6$ en $c=\frac{2}{5}$.

27a $\quad f(x)=(x^2-5x)^2=(x^2-5x)(x^2-5x)\Rightarrow f'(x)=(2x-5)(x^2-5x)+(x^2-5x)(2x-5)=2(x^2-5x)(2x-5)$.

27b $\quad f(x)=(x^2-5x)(x^2-5x)\Rightarrow f'(x)=[x^2-5x]'\cdot(x^2-5x)+(x^2-5x)\cdot[x^2-5x]'=2(x^2-5x)\cdot[x^2-5x]'$.

28 $\quad$ De tabel van $y_3=h(x)$ valt samen met de tabel van $y_2=g'(x)$ (de hellingfunctie van $g$).




Vergeet niet de ketting nog eens extra te differentiëren.

29a $\quad f(x)=\sqrt{x^2+4}=(x^2+4)^{\frac{1}{2}}\Rightarrow f'(x)=\frac{1}{2}(x^2+4)^{-\frac{1}{2}}\cdot 2x=x\cdot\frac{1}{(x^2+4)^{\frac{1}{2}}}=\frac{x}{\sqrt{x^2+4}}$.

Of korter: $f(x)=\sqrt{x^2+4}\Rightarrow f'(x)=\frac{1}{2\cdot\sqrt{x^2+4}}\cdot 2x=\frac{x}{\sqrt{x^2+4}}$. $\quad$ Leer van buiten: $[\sqrt{x}]'=\frac{1}{2\cdot\sqrt{x}}$.

29b $\quad g(x)=(2x^4+x^2)^3\Rightarrow g'(x)=3(2x^4+x^2)^2\cdot(8x^3+2x)$.

29c $\quad h(x)=\sqrt[3]{x^3+3x}=(x^3+3x)^{\frac{1}{3}}\Rightarrow h'(x)=\frac{1}{3}(x^3+3x)^{-\frac{2}{3}}\cdot(3x^2+3)=\frac{3x^2+3}{3\cdot(x^3+3x)^{\frac{2}{3}}}=\frac{x^2+1}{\sqrt[3]{(x^3+3x)^2}}$.

29d $\quad j(x)=(2x+1)^{-2}\Rightarrow j'(x)=-2(2x+1)^{-3}\cdot 2=-4(2x+1)^{-3}=\frac{-4}{(2x+1)^3}$.

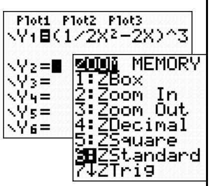
30a $\quad f(x)=\frac{1}{(3x+1)^2}=(3x+1)^{-2}\Rightarrow f'(x)=-2(3x+1)^{-3}\cdot 3=-6(3x+1)^{-3}=\frac{-6}{(3x+1)^3}$.

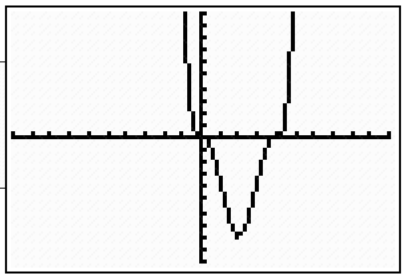
30b $\quad g(x)=\frac{1}{\sqrt{4x-1}}=\frac{1}{(4x-1)^{\frac{1}{2}}}=(4x-1)^{-\frac{1}{2}}\Rightarrow g'(x)=-\frac{1}{2}(4x-1)^{-1\frac{1}{2}}\cdot 4=-2\cdot\frac{1}{(4x-1)^{1\frac{1}{2}}}=\frac{-2}{(4x-1)\cdot\sqrt{4x-1}}$.

30c $\quad h(x)=(x^2+4)\cdot\sqrt{x^2+4}=(x^2+4)\cdot(x^2+4)^{\frac{1}{2}}=(x^2+4)^{1\frac{1}{2}}\Rightarrow h'(x)=1\frac{1}{2}(x^2+4)^{\frac{1}{2}}\cdot 2x=3x\cdot\sqrt{x^2+4}$.

30d $\quad j(x)=\frac{x^2+4}{\sqrt{x^2+4}}=\frac{x^2+4}{(x^2+4)^{\frac{1}{2}}}=(x^2+4)^{\frac{1}{2}}\Rightarrow j'(x)=\frac{1}{2}(x^2+4)^{-\frac{1}{2}}\cdot 2x=x\cdot\frac{1}{(x^2+4)^{\frac{1}{2}}}=\frac{x}{\sqrt{x^2+4}}$.

31a $\quad$ Maak een schets van de plot hiernaast.



31b $\quad f(x)=(\frac{1}{2}x^2-2x)^3\Rightarrow f'(x)=3(\frac{1}{2}x^2-2x)^2\cdot(x-2)$.

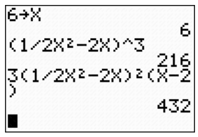
$f'(x)=0\Rightarrow 3(\frac{1}{2}x^2-2x)^2\cdot(x-2)=0$

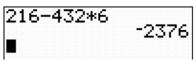
$\frac{1}{2}x^2-2x=0\vee x-2=0$

$x^2-4x=0\vee x=2$

$x(x-4)=0\vee x=2$

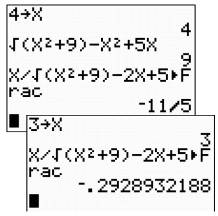
$x=0\vee x=4\vee x=2$




31c $\quad y_A=f(6)=216$ en $rc_{raaklijn}=f'(6)=432$.

$l$: $y=432x+b$ door $A(6,216)\Rightarrow 432\cdot 6+b=216\Rightarrow b=216-432\cdot 6=-2376$. Dus $l$: $y=432x-2376$.




32a $f(x)=\sqrt{x^2+9}-x^2+5x \Rightarrow f'(x)=\frac{1}{2\cdot\sqrt{x^2+9}}\cdot 2x-2x+5=\frac{x}{\sqrt{x^2+9}}-2x+5.$ $\quad [\sqrt{x}]'=\frac{1}{2\cdot\sqrt{x}}.$

$y_A=f(4)=9$ en $rc_{raaklijn}=f'(4)=-\frac{11}{5}.$

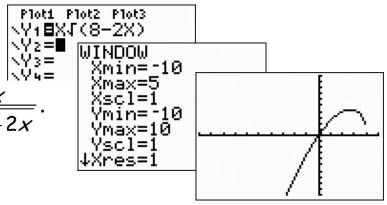
$k$: $y=-\frac{11}{5}x+b$ door $A(4,9)\Rightarrow -\frac{11}{5}\cdot 4+b=9\Rightarrow b=9+\frac{44}{5}=\frac{89}{5}$. Dus $k$: $y=-\frac{11}{5}x+\frac{89}{5}.$

32b $f'(3)=\frac{3}{\sqrt{3^2+9}}-2\cdot 3+5=\frac{3}{\sqrt{18}}-1\neq 0\Rightarrow f$ heeft geen extreme waarde voor $x=3$.

33 $f(x)=x\cdot\sqrt{2x+1}\Rightarrow f'(x)=1\cdot\sqrt{2x+1}+x\cdot\frac{1}{2\cdot\sqrt{2x+1}}\cdot 2=\sqrt{2x+1}+\frac{x}{\sqrt{2x+1}}$. (zie **Theorie B**)

34a $f(x)=x\cdot\sqrt{3x+1}\Rightarrow f'(x)=1\cdot\sqrt{3x+1}+x\cdot\frac{1}{2\cdot\sqrt{3x+1}}\cdot 3=\sqrt{3x+1}+\frac{3x}{2\cdot\sqrt{3x+1}}.$

34b $g(x)=x\cdot(3x+1)^3\Rightarrow g'(x)=1\cdot(3x+1)^3+x\cdot 3(3x+1)^2\cdot 3=(3x+1)^3+9x(3x+1)^2.$



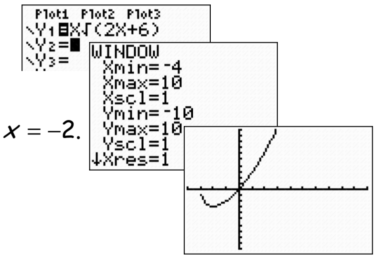

35a $f(x)=x\cdot\sqrt{8-2x}$ (BV: $8-2x\geq 0\Rightarrow -2x\geq -8\Rightarrow x\leq 4$) $\Rightarrow D_f=\langle\leftarrow, 4].$

35b $f(x)=x\cdot\sqrt{8-2x}\Rightarrow f'(x)=1\cdot\sqrt{8-2x}+x\cdot\frac{1}{2\cdot\sqrt{8-2x}}\cdot -2=\sqrt{8-2x}-\frac{x}{\sqrt{8-2x}}.$

35c $f'(x)=0\Rightarrow\sqrt{8-2x}-\frac{x}{\sqrt{8-2x}}=0\Rightarrow\frac{\sqrt{8-2x}}{1}=\frac{x}{\sqrt{8-2x}}\Rightarrow 8-2x=x\cdot 1.$

35d $-3x=-8\Rightarrow x_{top}=\frac{-8}{-3}=\frac{8}{3}$ en $y_{top}=f(\frac{8}{3})=\frac{8}{3}\cdot\sqrt{8-2\cdot\frac{8}{3}}=\frac{8}{3}\cdot\sqrt{\frac{24}{3}-\frac{16}{3}}=\frac{8}{3}\cdot\sqrt{\frac{8}{3}}$. Dus de top is $(\frac{8}{3}, \frac{8}{3}\sqrt{\frac{8}{3}})$.

35e Het extreem in de top is een maximum (zie een plot) $\Rightarrow B_f=\langle\leftarrow, \frac{8}{3}\sqrt{\frac{8}{3}}].$




36a $f(x)=x\cdot\sqrt{2x+6}$ (BV: $2x+6\geq 0\Rightarrow 2x\geq -6\Rightarrow x\geq -3$) $\Rightarrow D_f=[-3, \rightarrow\rangle.$

36b $f(x)=x\cdot\sqrt{2x+6}\Rightarrow f'(x)=1\cdot\sqrt{2x+6}+x\cdot\frac{1}{2\cdot\sqrt{2x+6}}\cdot 2=\sqrt{2x+6}+\frac{x}{\sqrt{2x+6}}.$

$f'(x)=0\Rightarrow\sqrt{2x+6}+\frac{x}{\sqrt{2x+6}}=0\Rightarrow\frac{\sqrt{2x+6}}{1}=\frac{-x}{\sqrt{2x+6}}\Rightarrow 2x+6=-x\Rightarrow 3x=-6\Rightarrow x=-2.$

$x_{top}=-2$ en $y_{top}=f(-2)=-2\cdot\sqrt{-4+6}=-2\cdot\sqrt{2}$. Dus de top is $(-2, -2\sqrt{2})$.

36c Het extreem in de top is een minimum (zie een plot) $\Rightarrow B_f=[-2\sqrt{2}, \rightarrow\rangle.$

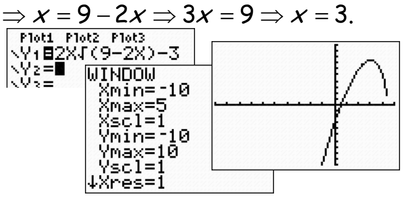
37a $f(x)=2x\cdot\sqrt{9-2x}-3\Rightarrow f'(x)=2\cdot\sqrt{9-2x}+2x\cdot\frac{1}{2\cdot\sqrt{9-2x}}\cdot -2-0=2\sqrt{9-2x}-\frac{2x}{\sqrt{9-2x}}.$

$y_A=f(0)=0\cdot\sqrt{9}-3=-3$ en $rc_{raaklijn}=f'(0)=2\cdot\sqrt{9}-\frac{0}{\sqrt{9}}=2\cdot 3-0=6.$

$k$: $y=6x+b$ door $A(0,-3)\Rightarrow 6\cdot 0+b=-3\Rightarrow b=-3+0=-3$. Dus $k$: $y=6x-3$.

37b $f'(x)=0\Rightarrow 2\cdot\sqrt{9-2x}-\frac{2x}{\sqrt{9-2x}}=0\Rightarrow\frac{2\cdot\sqrt{9-2x}}{1}=\frac{2x}{\sqrt{9-2x}}\Rightarrow 2x=2(9-2x)\Rightarrow x=9-2x\Rightarrow 3x=9\Rightarrow x=3.$

$x_{top}=3$ en het maximum (zie een plot) is $y_{top}=f(3)=6\cdot\sqrt{9-6}-3=6\sqrt{3}-3.$




37c $D_f=\langle\leftarrow, 4\frac{1}{2}]$ (BV: $9-2x\geq 0\Rightarrow -2x\geq -9\Rightarrow x\leq 4\frac{1}{2}$) en $B_f=\langle\leftarrow, 6\sqrt{3}]$ (zie 37b).

38a De hoogtelijn $CD$ deelt $AB$ doormidden $\Rightarrow AD=DB=1$.

De stelling van Pythagoras in $\Delta ADC$ geeft $CD=\sqrt{AC^2-AD^2}=\sqrt{2^2-1^2}=\sqrt{3}.$

38b In $\Delta ADC$: $\sin\angle DAC=\sin 60°=\frac{\text{overst. rz.}}{\text{schuine z.}}=\frac{DC}{AC}=\frac{\sqrt{3}}{2}=\frac{1}{2}\sqrt{3}$ en $\cos\angle DAC=\cos 60°=\frac{\text{aanl. rz.}}{\text{schuine z.}}=\frac{AD}{AC}=\frac{1}{2}.$

38c In $\Delta ADC$: $\sin\angle DCA=\sin 30°=\frac{\text{overst. rz.}}{\text{schuine z.}}=\frac{AD}{AC}=\frac{1}{2}$ en $\cos\angle DCA=\cos 30°=\frac{\text{aanl. rz.}}{\text{schuine z.}}=\frac{DC}{AC}=\frac{\sqrt{3}}{2}=\frac{1}{2}\sqrt{3}.$

38d De stelling van Pythagoras in $\Delta ABC$ (fig. 12.12) geeft $AC=\sqrt{AB^2+BC^2}=\sqrt{1^2+1^2}=\sqrt{2}.$

In $\Delta ABC$ (fig. 12.12): $\sin\angle BAC=\sin 45°=\frac{BC}{AC}=\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}=\frac{1}{2}\sqrt{2}$ en $\cos\angle BAC=\cos 45°=\frac{AB}{AC}=\frac{1}{\sqrt{2}}=\frac{1}{2}\sqrt{2}.$

39a $\sin(\frac{3}{4}\pi) = \frac{1}{2}\sqrt{2}.$ 39c $\sin(1\frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}.$ 39e $\cos(1\frac{1}{3}\pi) = -\frac{1}{2}.$

39b $\cos(\frac{7}{6}\pi) = -\frac{1}{2}\sqrt{3}.$ 39d $\cos(\frac{5}{3}\pi) = \frac{1}{2}.$ 39f $\sin(-\frac{1}{4}\pi) = \sin(\frac{7}{4}\pi) = -\frac{1}{2}\sqrt{2}.$

40a $\sin(\alpha) = \frac{1}{2}\sqrt{3}$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = \frac{1}{3}\pi \vee \alpha = \frac{2}{3}\pi.$

40b $\cos(\alpha) = -\frac{1}{2}$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = \frac{2}{3}\pi \vee \alpha = 1\frac{1}{3}\pi.$

40c $\sin(\alpha) = -\frac{1}{2}\sqrt{2}$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = 1\frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi.$

40d $\cos(\alpha) = 0$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = \frac{1}{2}\pi \vee \alpha = 1\frac{1}{2}\pi.$

40e $\cos(\alpha) = \frac{1}{2}\sqrt{3}$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = \frac{1}{6}\pi \vee \alpha = 1\frac{5}{6}\pi.$

40f $\cos(\alpha) = \frac{1}{2}\sqrt{2}$ $(0 \le \alpha \le 2\pi) \Rightarrow \alpha = \frac{1}{4}\pi \vee \alpha = 1\frac{3}{4}\pi.$

41 $\ldots,\ -6\frac{1}{2}\pi,\ -5\frac{1}{2}\pi,\ -4\frac{1}{2}\pi,\ -3\frac{1}{2}\pi,\ -2\frac{1}{2}\pi,\ -1\frac{1}{2}\pi,\ -\frac{1}{2}\pi,\ \frac{1}{2}\pi,\ 1\frac{1}{2}\pi,\ 2\frac{1}{2}\pi,\ 3\frac{1}{2}\pi,\ 4\frac{1}{2}\pi,\ 5\frac{1}{2}\pi,\ 6\frac{1}{2}\pi,\ \ldots$

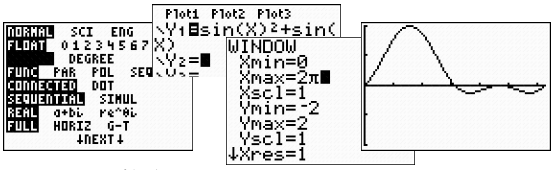
42a
$\sin(3x - \frac{1}{2}\pi) = 0$
$3x - \frac{1}{2}\pi = k \cdot \pi$
$3x = \frac{1}{2}\pi + k \cdot \pi$
$x = \frac{1}{6}\pi + k \cdot \frac{1}{3}\pi.$

42b
$\cos(\frac{1}{2}x - \frac{1}{6}\pi) = 0$
$\frac{1}{2}x - \frac{1}{6}\pi = \frac{1}{2}\pi + k \cdot \pi$
$\frac{1}{2}x = \frac{2}{3}\pi + k \cdot \pi$
$x = \frac{4}{3}\pi + k \cdot 2\pi.$

42c
$\sin^2(x) = \sin(x)$
$\sin^2(x) - \sin(x) = 0$
$\sin(x) \cdot (\sin(x) - 1) = 0$
$\sin(x) = 0 \vee \sin(x) = 1$
$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi.$

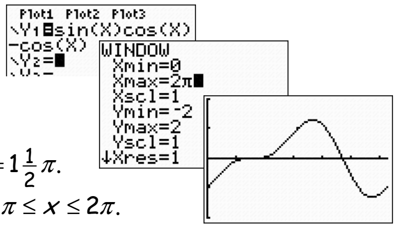
42d
$\cos^2(2x) + \cos(2x) = 0$
$\cos(2x) \cdot (\cos(2x) + 1) = 0$
$\cos(2x) = 0 \vee \cos(2x) = -1$
$x = \frac{1}{2}\pi + k \cdot \pi \vee x = k \cdot 2\pi.$

43
$f(x) = \sin^2(x) + \sin(x) = 0$
$\sin(x) \cdot (\sin(x) + 1) = 0$
$\sin(x) = 0 \vee \sin(x) = -1$
$x = k \cdot \pi \vee x = -\frac{1}{2}\pi + k \cdot 2\pi.$



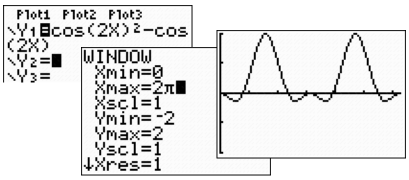

$f(x) = 0$ (met $0 \le x \le 2\pi$) $\Rightarrow x = 0 \vee x = \pi \vee x = 1\frac{1}{2}\pi \vee x = 2\pi.$ $f(x) \le 0$ (zie een plot) $\Rightarrow x = 0 \vee \pi \le x \le 2\pi.$

44a
$\sin(x)\cos(x) - \cos(x) = 0$
$\cos(x) \cdot (\sin(x) - 1) = 0$
$\cos(x) = 0 \vee \sin(x) = 1$
$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi.$




$\sin(x)\cos(x) - \cos(x) = 0$ (met $0 \le x \le 2\pi$) $\Rightarrow x = \frac{1}{2}\pi \vee x = 1\frac{1}{2}\pi.$
$\sin(x)\cos(x) - \cos(x) \le 0$ (zie een plot) $\Rightarrow 0 \le x \le \frac{1}{2}\pi \vee 1\frac{1}{2}\pi \le x \le 2\pi.$

44b
$\cos^2(2x) - \cos(2x) = 0$
$\cos(2x) \cdot (\cos(2x) - 1) = 0$
$\cos(2x) = 0 \vee \cos(2x) = 1$
$2x = \frac{1}{2}\pi + k \cdot \pi \vee 2x = k \cdot 2\pi$
$x = \frac{1}{4}\pi + k \cdot \frac{1}{2}\pi \vee x = k \cdot \pi.$




$\cos^2(2x) - \cos(2x) = 0$ (met $0 \le x \le 2\pi$) $\Rightarrow x = 0 \vee x = \frac{1}{4}\pi \vee x = \frac{3}{4}\pi \vee x = \pi \vee x = 1\frac{1}{4}\pi \vee x = 1\frac{3}{4}\pi \vee x = 2\pi.$
$\cos^2(2x) - \cos(2x) \ge 0$ (zie een plot) $\Rightarrow x = 0 \vee \frac{1}{4}\pi \le x \le \frac{3}{4}\pi \vee x = \pi \vee 1\frac{1}{4}\pi \le x \le 1\frac{3}{4}\pi \vee x = 2\pi.$

45a $\sin(\frac{1}{6}\pi) = \frac{1}{2} \Rightarrow x = \frac{1}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}.$

45b $2\frac{1}{6}\pi$ en $4\frac{1}{6}\pi$ liggen op dezelfde plaats als $\frac{1}{6}\pi$ op de eenheidscirkel. (precies één of twee rondgangen verder)

45c $\sin(\frac{5}{6}\pi) = \frac{1}{2} \Rightarrow x = \frac{5}{6}\pi$ is een oplossing van $\sin(x) = \frac{1}{2}.$

45d $2\frac{5}{6}\pi$ en $-1\frac{1}{6}\pi$ liggen op dezelfde plaats als $\frac{5}{6}\pi$ op de eenheidscirkel. (precies één rondgang verder of terug)

46a
$2\sin(\frac{1}{2}x) = 1$
$\sin(\frac{1}{2}x) = \frac{1}{2}$
$\frac{1}{2}x = \frac{1}{6}\pi + k \cdot 2\pi \vee \frac{1}{2}x = \frac{5}{6}\pi + k \cdot 2\pi$ (keer 2)
$x = \frac{1}{3}\pi + k \cdot 4\pi \vee x = \frac{5}{3}\pi + k \cdot 4\pi.$

46b
$2\cos(x - \frac{1}{3}\pi) = 1$
$\cos(x - \frac{1}{3}\pi) = \frac{1}{2}$
$x - \frac{1}{3}\pi = \frac{1}{3}\pi + k \cdot 2\pi \vee x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi$
$x = \frac{2}{3}\pi + k \cdot 2\pi \vee x = k \cdot 2\pi.$

**46c** $2\sin(2x-\frac{1}{4}\pi)=-\sqrt{3}$

$\sin(2x-\frac{1}{4}\pi)=-\frac{1}{2}\sqrt{3}$

$2x-\frac{1}{4}\pi=\frac{4}{3}\pi+k\cdot 2\pi \vee 2x-\frac{1}{4}\pi=-\frac{1}{3}\pi+k\cdot 2\pi$

$2x=\frac{19}{12}\pi+k\cdot 2\pi \vee 2x=-\frac{1}{12}\pi+k\cdot 2\pi$

$x=\frac{19}{24}\pi+k\cdot \pi \vee x=-\frac{1}{24}\pi+k\cdot \pi.$

**46d** $2\cos(3x-\pi)=-1$

$\cos(3x-\pi)=-\frac{1}{2}$

$3x-\pi=\frac{2}{3}\pi+k\cdot 2\pi \vee 3x-\pi=-\frac{2}{3}\pi+k\cdot 2\pi$

$3x=\frac{5}{3}\pi+k\cdot 2\pi \vee 3x=\frac{1}{3}\pi+k\cdot 2\pi$

$x=\frac{5}{9}\pi+k\cdot \frac{2}{3}\pi \vee x=\frac{1}{9}\pi+k\cdot \frac{2}{3}\pi.$

**47a** $2\sin(2x-\frac{1}{6}\pi)=\sqrt{2}$

$\sin(2x-\frac{1}{6}\pi)=\frac{1}{2}\sqrt{2}$

$2x-\frac{1}{6}\pi=\frac{1}{4}\pi+k\cdot 2\pi \vee 2x-\frac{1}{6}\pi=\frac{3}{4}\pi+k\cdot 2\pi$

$2x=\frac{5}{12}\pi+k\cdot 2\pi \vee 2x=\frac{11}{12}\pi+k\cdot 2\pi$

$x=\frac{5}{24}\pi+k\cdot \pi \vee x=\frac{11}{24}\pi+k\cdot \pi.$ (met $x$ op $[0, 2\pi]$)

$x=\frac{5}{24}\pi \vee x=1\frac{5}{24}\pi \vee x=\frac{11}{24}\pi \vee x=1\frac{11}{24}\pi.$

**47c** $\sin(\frac{2}{3}x)=-\frac{1}{2}\sqrt{2}$

$\frac{2}{3}x=-\frac{1}{4}\pi+k\cdot 2\pi \vee \frac{2}{3}x=\frac{5}{4}\pi+k\cdot 2\pi$ (keer $\frac{3}{2}$)

$x=-\frac{3}{8}\pi+k\cdot 3\pi \vee x=\frac{15}{8}\pi+k\cdot 3\pi.$ (met $x$ op $[0, 2\pi]$)

$x=\frac{15}{8}\pi.$

**47b** $2\cos(3x-\frac{1}{2}\pi)=\sqrt{3}$

$\cos(3x-\frac{1}{2}\pi)=\frac{1}{2}\sqrt{3}$

$3x-\frac{1}{2}\pi=\frac{1}{6}\pi+k\cdot 2\pi \vee 3x-\frac{1}{2}\pi=-\frac{1}{6}\pi+k\cdot 2\pi$

$3x=\frac{2}{3}\pi+k\cdot 2\pi \vee 3x=\frac{1}{3}\pi+k\cdot 2\pi$

$x=\frac{2}{9}\pi+k\cdot \frac{2}{3}\pi \vee x=\frac{1}{9}\pi+k\cdot \frac{2}{3}\pi.$ (met $x$ op $[0, 2\pi]$)

$x=\frac{2}{9}\pi \vee x=\frac{8}{9}\pi \vee x=\frac{14}{9}\pi \vee x=\frac{1}{9}\pi \vee x=\frac{7}{9}\pi \vee x=\frac{13}{9}\pi.$

**47d** $\cos(\frac{1}{2}x)=-\frac{1}{2}\sqrt{3}$

$\frac{1}{2}x=\frac{5}{6}\pi+k\cdot 2\pi \vee \frac{1}{2}x=-\frac{5}{6}\pi+k\cdot 2\pi$ (keer 2)

$x=\frac{5}{3}\pi+k\cdot 4\pi \vee x=-\frac{5}{3}\pi+k\cdot 4\pi.$ (met $x$ op $[0, 2\pi]$)

$x=\frac{5}{3}\pi.$

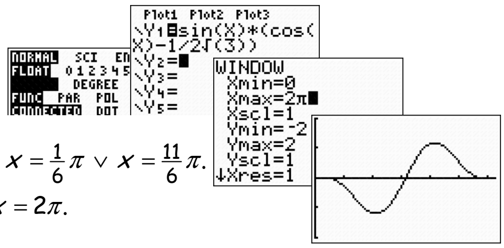
**48a** $\sin(x)\cdot\left(\cos(x)-\frac{1}{2}\sqrt{3}\right)=0$

$\sin(x)=0 \vee \cos(x)=\frac{1}{2}\sqrt{3}$

$x=k\cdot\pi \vee x=\frac{1}{6}\pi+k\cdot 2\pi \vee x=-\frac{1}{6}\pi+k\cdot 2\pi.$

$\sin(x)\cdot\left(\cos(x)-\frac{1}{2}\sqrt{3}\right)=0$ (met $0\le x\le 2\pi$) $\Rightarrow x=0 \vee x=\pi \vee x=2\pi \vee x=\frac{1}{6}\pi \vee x=\frac{11}{6}\pi.$

$\sin(x)\cdot\left(\cos(x)-\frac{1}{2}\sqrt{3}\right)\ge 0$ (zie een plot) $\Rightarrow 0\le x\le \frac{1}{6}\pi \vee \pi\le x\le \frac{11}{6}\pi \vee x=2\pi.$




**48b** $\sin^2(x)-1\frac{1}{2}\sin(x)-1=0$

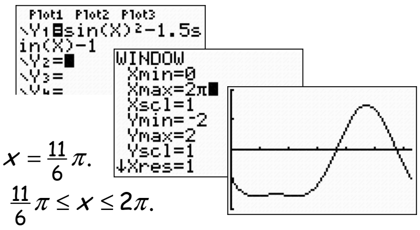
$\left(\sin(x)-2\right)\cdot\left(\sin(x)+\frac{1}{2}\right)=0$

$\sin(x)=2 \vee \sin(x)=-\frac{1}{2}$

$x=$ kan niet $\vee x=\frac{7}{6}\pi+k\cdot 2\pi \vee x=-\frac{1}{6}\pi+k\cdot 2\pi.$

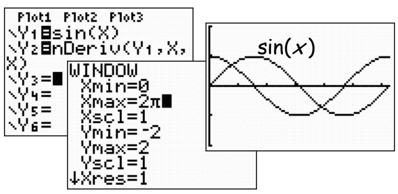
$\sin^2(x)-1\frac{1}{2}\sin(x)-1=0$ (met $0\le x\le 2\pi$) $\Rightarrow x=\frac{7}{6}\pi \vee x=\frac{11}{6}\pi.$

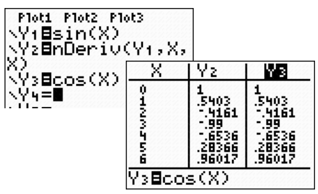
$\sin^2(x)-1\frac{1}{2}\sin(x)-1\le 0$ (zie een plot) $\Rightarrow 0\le x\le \frac{7}{6}\pi \vee \frac{11}{6}\pi\le x\le 2\pi.$




**49a** Zie de eerste drie schermen hiernaast.

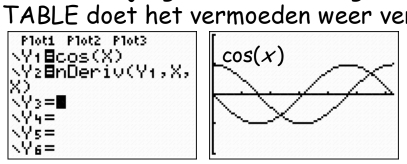
**49b** Vermoedelijk: $f(x)=\sin(x)\Rightarrow f'(x)=\cos(x).$
TABLE doet het vermoeden versterken.

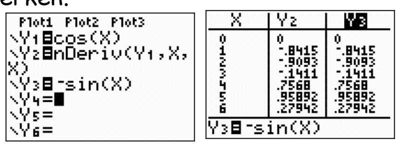



| X | Y2 | Y3 |
|---|---|---|
| 0 | 1 | 1 |
| 1 | .5403 | .5403 |
| 2 | -.4161 | -.4161 |
| 3 | -.99 | -.99 |
| 4 | -.6536 | -.6536 |
| 5 | .28366 | .28366 |
| 6 | .96017 | .96017 |

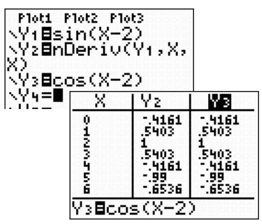
**49c** Zie de eerste twee schermen hieronder.
Vermoedelijk: $g(x)=\cos(x)\Rightarrow g'(x)=-\sin(x).$
TABLE doet het vermoeden weer versterken.



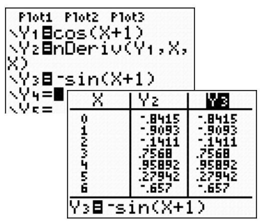




| X | Y2 | Y3 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | -.8415 | -.8415 |
| 2 | -.9093 | -.9093 |
| 3 | -.1411 | -.1411 |
| 4 | .7568 | .7568 |
| 5 | .95892 | .95892 |
| 6 | .27942 | .27942 |

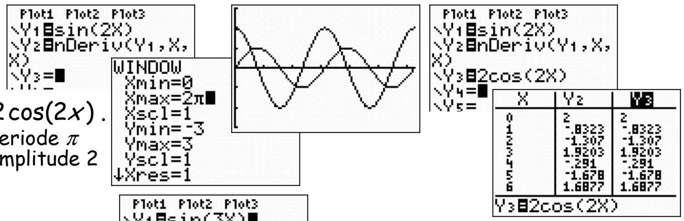
**49d** Vermoedelijk: $h(x)=\sin(\boxed{x-2})\Rightarrow h'(x)=\cos(x-2)\cdot 1=\cos(x-2).$
TABLE doet het vermoeden alleen maar versterken.
Zie de eerste twee schermen hiernaast.
Vermoedelijk: $j(x)=\cos(\boxed{x+1})\Rightarrow j'(x)=-\sin(x+1)\cdot 1=-\sin(x+1).$
TABLE doet het vermoeden versterken.
Zie de laatste twee schermen hiernaast.



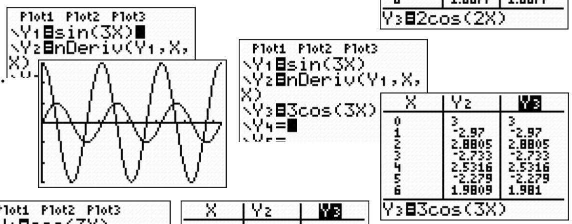

| X | Y2 | Y3 |
|---|---|---|
| 0 | -.4161 | -.4161 |
| 1 | .5403 | .5403 |
| 2 | 1 | 1 |
| 3 | .5403 | .5403 |
| 4 | -.4161 | -.4161 |
| 5 | -.99 | -.99 |
| 6 | -.6536 | -.6536 |



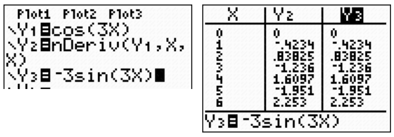

| X | Y2 | Y3 |
|---|---|---|
| 0 | -.8415 | -.8415 |
| 1 | -.9093 | -.9093 |
| 2 | -.1411 | -.1411 |
| 3 | .7568 | .7568 |
| 4 | .95892 | .95892 |
| 5 | .27942 | .27942 |
| 6 | -.657 | -.657 |




50ac Zie de eerste drie schermen hiernaast.
Vermoedelijk $f(x) = \sin(2x) \Rightarrow f'(x) = \cos(2x) \cdot 2 = 2\cos(2x)$.
periode $\frac{2\pi}{2} = \pi$, amplitude 1 — periode $\pi$, amplitude 2
Controle met TABLE doet het vermoeden versterken.




50bc Zie de eerste twee schermen hiernaast.
Vermoedelijk $g(x) = \sin(3x) \Rightarrow g'(x) = \cos(3x) \cdot 3 = 3\cos(3x)$.
periode $\frac{2\pi}{3} = \frac{2}{3}\pi$, amplitude 1 — periode $\frac{2}{3}\pi$, amplitude 3
Controle met TABLE doet het vermoeden versterken.




51 Vermoedelijk $k(x) = \cos(3x) \Rightarrow k'(x) = -\sin(3x) \cdot 3 = -3\sin(3x)$.
Controle met TABLE doet het vermoeden versterken.

52a $f(x) = \cos(2x) \Rightarrow f'(x) = -\sin(2x) \cdot 2 = -2\sin(2x)$.

52b $g(x) = x \cdot \cos(x) \Rightarrow g'(x) = 1 \cdot \cos(x) + x \cdot -\sin(x) = \cos(x) - x\sin(x)$.

52c $h(x) = 3 + 4\sin(2x - \frac{1}{3}\pi) \Rightarrow h'(x) = 0 + 4\cos(2x - \frac{1}{3}\pi) \cdot 2 = 8\cos(2x - \frac{1}{3}\pi)$.

52d $j(x) = 10 + 16\sin(\frac{1}{2}(x-1)) = 10 + 16\sin(\frac{1}{2}x - \frac{1}{2}) \Rightarrow j'(x) = 0 + 16\cos(\frac{1}{2}x - \frac{1}{2}) \cdot \frac{1}{2} = 8\cos(\frac{1}{2}x - \frac{1}{2})$.
Of $j(x) = 10 + 16\sin(\frac{1}{2}(x-1)) \Rightarrow j'(x) = 0 + 16\cos(\frac{1}{2}x - \frac{1}{2}) \cdot \frac{1}{2} \cdot 1 = 8\cos(\frac{1}{2}x - \frac{1}{2})$.

53a $f(x) = \sin(ax + b) \Rightarrow f'(x) = \cos(ax + b) \cdot a = a\cos(ax + b)$.

53b $g(x) = \cos(ax + b) \Rightarrow g'(x) = -\sin(ax + b) \cdot a = -a\sin(ax + b)$.

54ab I: $f(x) = x\sin(2x) \Rightarrow f'(x) = 1 \cdot \sin(2x) + x \cdot \cos(2x) \cdot 2 = \sin(2x) + 2x\cos(2x)$. (mijn voorkeur)
II: $f(x) = x\sin(2x) \Rightarrow f'(x) = 1 \cdot \sin(2x) + x \cdot 2\cos(2x) = \sin(2x) + 2x\cos(2x)$.

55a $f(x) = x\cos(2x) \Rightarrow f'(x) = 1 \cdot \cos(2x) + x \cdot -\sin(2x) \cdot 2 = \cos(2x) - 2x\sin(2x)$.

55b $g(x) = x^2\sin(3x) \Rightarrow g'(x) = 2x \cdot \sin(3x) + x^2 \cdot \cos(3x) \cdot 3 = 2x\sin(3x) + 3x^2\cos(3x)$.

55c $h(x) = 2x\sin(3x - 1) \Rightarrow h'(x) = 2 \cdot \sin(3x - 1) + 2x \cdot \cos(3x - 1) \cdot 3 = 2\sin(3x - 1) + 6x\cos(3x - 1)$.

55d $j(x) = 1 + 3x\cos(\frac{1}{2}x) \Rightarrow j'(x) = 0 + 3 \cdot \cos(\frac{1}{2}x) + 3x \cdot -\sin(\frac{1}{2}x) \cdot \frac{1}{2} = 3\cos(\frac{1}{2}x) - 1\frac{1}{2}x\sin(\frac{1}{2}x)$.

56ab I: $f(x) = \sin^2(x) = \sin(x) \cdot \sin(x) \Rightarrow f'(x) = \cos(x) \cdot \sin(x) + \sin(x) \cdot \cos(x) = 2\sin(x) \cdot \cos(x)$.
II: $f(x) = \sin^2(x) = (\sin(x))^2 \Rightarrow f'(x) = 2\sin(x) \cdot \cos(x)$. (mijn voorkeur, maar bepaal je eigen voorkeur).

57a $f(x) = \cos^2(x) = (\cos(x))^2 \Rightarrow f'(x) = 2\cos(x) \cdot -\sin(x) = -2\sin(x) \cdot \cos(x)$.

57b $g(x) = 2\sin^2(x) = 2(\sin(x))^2 \Rightarrow g'(x) = 2 \cdot 2\sin(x) \cdot \cos(x) = 4\sin(x) \cdot \cos(x)$.

57c $h(x) = 1 + 2\cos^2(x) = 1 + 2(\cos(x))^2 \Rightarrow h'(x) = 0 + 2 \cdot 2\cos(x) \cdot -\sin(x) = -4\sin(x) \cdot \cos(x)$.

57d $j(x) = x + 3\sin^2(x) = x + 3(\sin(x))^2 \Rightarrow j'(x) = 1 + 3 \cdot 2\sin(x) \cdot \cos(x) = 1 + 6\sin(x) \cdot \cos(x)$.

58a $f(x) = \sin^3(x) = (\sin(x))^3 \Rightarrow f'(x) = 3(\sin(x))^2 \cdot \cos(x) = 3\sin^2(x) \cdot \cos(x)$.

58b $g(x) = x\sin^2(x) = x \cdot (\sin(x))^2 \Rightarrow g'(x) = 1 \cdot \sin^2(x) + x \cdot 2\sin(x) \cdot \cos(x) = \sin^2(x) + 2x\sin(x) \cdot \cos(x)$.

58c $h(x) = \sqrt{2 + \sin(x)} \Rightarrow h'(x) = \frac{1}{2 \cdot \sqrt{2 + \sin(x)}} \cdot \cos(x) = \frac{\cos(x)}{2 \cdot \sqrt{2 + \sin(x)}}$. Gebruik: $[\sqrt{x}]' = \frac{1}{2 \cdot \sqrt{x}}$.

58d $j(x) = 2x \cdot \cos(x^2) \Rightarrow j'(x) = 2 \cdot \cos(x^2) + 2x \cdot -\sin(x^2) \cdot 2x = 2\cos(x^2) - 4x^2\sin(x^2)$.

59a $f(x)=\cos^2(x)-\cos(x)=\left(\boxed{\cos(x)}\right)^2-\cos(x)\Rightarrow f'(x)=2\cos(x)\cdot-\sin(x)+\sin(x)=-2\sin(x)\cdot\cos(x)+\sin(x).$

$f'(x)=0\Rightarrow -2\sin(x)\cdot\cos(x)+\sin(x)=0$

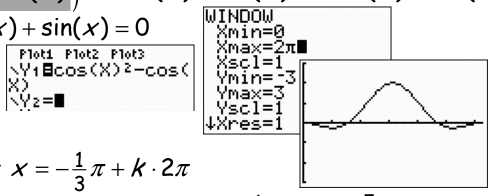
$\sin(x)\cdot\left(-2\cos(x)+1\right)=0$

$\sin(x)=0\ \vee\ -2\cos(x)=-1$

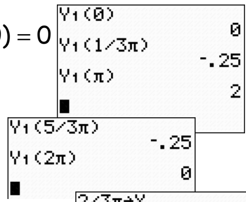
$x=k\cdot\pi\ \vee\ \cos(x)=\frac{1}{2}$

$x=k\cdot\pi\ \vee\ x=\frac{1}{3}\pi+k\cdot 2\pi\ \vee\ x=-\frac{1}{3}\pi+k\cdot 2\pi$

$x$ op $[0,2\pi]$ geeft $x=0\vee x=\pi\vee x=2\pi\vee x=\frac{1}{3}\pi\vee x=\frac{5}{3}\pi.$

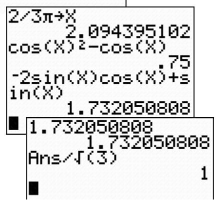



max. (zie plot) $f(0)=0$
min. $f(\frac{1}{3}\pi)=-\frac{1}{4}$
max. $f(\pi)=2$
min. $f(\frac{5}{3}\pi)=-\frac{1}{4}$
max. $f(2\pi)=0.$




59b $y_A=f(\frac{2}{3}\pi)=\frac{3}{4}$ en $rc_{raaklijn}=f'(\frac{2}{3}\pi)=\sqrt{3}.$

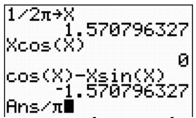
$k$: $y=\sqrt{3}\cdot x+b$ door $A(\frac{2}{3}\pi,\ \frac{3}{4})\Rightarrow\sqrt{3}\cdot\frac{2}{3}\pi+b=\frac{3}{4}\Rightarrow b=\frac{3}{4}-\frac{2}{3}\pi\sqrt{3}.$ Dus $k$: $y=x\sqrt{3}+\frac{3}{4}-\frac{2}{3}\pi\sqrt{3}.$




60a $f(x)=x\cdot\cos(x)\Rightarrow f'(x)=1\cdot\cos(x)+x\cdot-\sin(x)=\cos(x)-x\sin(x).$

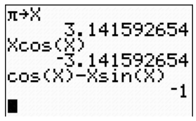
$y_A=f(\frac{1}{2}\pi)=\frac{1}{2}\pi\cdot 0=0$ en $rc_{raaklijn}=f'(\frac{2}{3}\pi)=0-\frac{1}{2}\pi\cdot 1=-\frac{1}{2}\pi.$

$k$: $y=-\frac{1}{2}\pi x+b$ door $A(\frac{1}{2}\pi,0)\Rightarrow-\frac{1}{2}\pi\cdot\frac{1}{2}\pi+b=0\Rightarrow b=\frac{1}{4}\pi^2.$ Dus $k$: $y=-\frac{1}{2}\pi x+\frac{1}{4}\pi^2.$

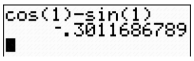



60b $y_B=f(\pi)=\pi\cdot-1=-\pi$ en $rc_{raaklijn}=f'(\pi)=-1-\pi\cdot 0=-1.$

$l$: $y=-x+b$ door $B(\pi,\ -\pi)\Rightarrow-\pi+b=-\pi\Rightarrow b=0\Rightarrow l$ gaat door de oorsprong.




60c $f'(1)=\cos(1)-1\cdot\sin(1)\neq 0\Rightarrow f$ heeft geen top voor $x=1.$




61a $f(x)=\sin^2(x)-\sqrt{3}\cdot\sin(x)=\left(\boxed{\sin(x)}\right)^2-\sqrt{3}\cdot\sin(x)\Rightarrow f'(x)=2\sin(x)\cdot\cos(x)-\sqrt{3}\cdot\cos(x).$

$f'(x)=0\Rightarrow 2\sin x\cdot\cos(x)-\sqrt{3}\cdot\cos(x)=0$

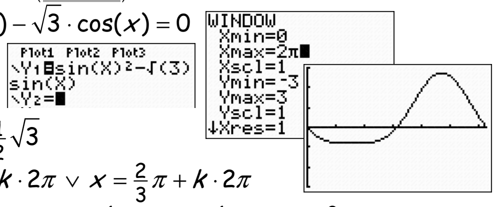
$\cos(x)\cdot\left(2\sin(x)-\sqrt{3}\right)=0$

$\cos(x)=0\ \vee\ 2\sin(x)=\sqrt{3}$

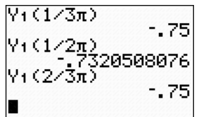
$x=\frac{1}{2}\pi+k\cdot\pi\ \vee\ \sin(x)=\frac{1}{2}\sqrt{3}$

$x=\frac{1}{2}\pi+k\cdot\pi\ \vee\ x=\frac{1}{3}\pi+k\cdot 2\pi\ \vee\ x=\frac{2}{3}\pi+k\cdot 2\pi$

$x$ op $[0,2\pi]$ geeft $x=\frac{1}{2}\pi\vee x=1\frac{1}{2}\pi\vee x=\frac{1}{3}\pi\vee x=\frac{2}{3}\pi.$




min. (zie plot) $f(\frac{1}{3}\pi)=-\frac{3}{4}$
max. $f(\frac{1}{2}\pi)=1^2-\sqrt{3}\cdot 1=1-\sqrt{3}$
min. $f(\frac{2}{3}\pi)=-\frac{3}{4}$
max. $f(1\frac{1}{2}\pi)=(-1)^2-\sqrt{3}\cdot-1=1+\sqrt{3}.$

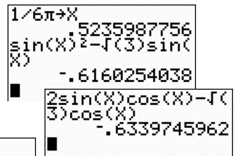



61b $y_A=f(\frac{1}{6}\pi)=\sin^2(\frac{1}{6}\pi)-\sqrt{3}\cdot\sin(\frac{1}{6}\pi)=(\frac{1}{2})^2-\sqrt{3}\cdot\frac{1}{2}=\frac{1}{4}-\frac{1}{2}\sqrt{3}$ en

$rc_{raaklijn}=f'(\frac{1}{6}\pi)=2\sin(\frac{1}{6}\pi)\cdot\cos(\frac{1}{6}\pi)-\sqrt{3}\cdot\cos(\frac{1}{6}\pi)=2\cdot\frac{1}{2}\cdot\frac{1}{2}\sqrt{3}-\sqrt{3}\cdot\frac{1}{2}\sqrt{3}=\frac{1}{2}\sqrt{3}-\frac{3}{2}.$

$k$: $y=(\frac{1}{2}\sqrt{3}-\frac{3}{2})x+b$ door $A(\frac{1}{6}\pi,\ \frac{1}{4}-\frac{1}{2}\sqrt{3})\Rightarrow(\frac{1}{2}\sqrt{3}-\frac{3}{2})\cdot\frac{1}{6}\pi+b=\frac{1}{4}-\frac{1}{2}\sqrt{3}.$

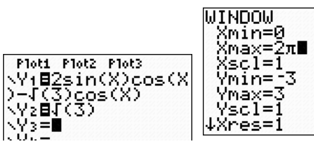
Dus $k$: $y=(\frac{1}{2}\sqrt{3}-\frac{3}{2})x+\frac{1}{4}-\frac{1}{2}\sqrt{3}-\frac{1}{6}\pi(\frac{1}{2}\sqrt{3}-\frac{3}{2}).$

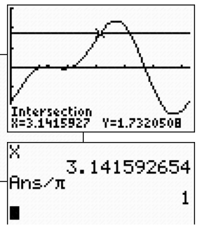


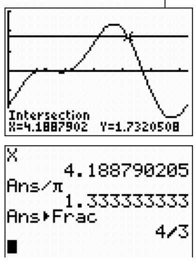

61c $f'(x)=\sqrt{3}\Rightarrow 2\sin(x)\cdot\cos(x)-\sqrt{3}\cdot\cos(x)=\sqrt{3}$

(niet algebraïsch op te lossen) intersect geeft

$x\approx 3{,}1415...=\pi\ \vee\ x\approx 4{,}1887...=\frac{4}{3}\pi.$




Controle: $f'(\pi)=2\sin(\pi)\cdot\cos(\pi)-\sqrt{3}\cdot\cos(\pi)=2\cdot 0\cdot-1-\sqrt{3}\cdot-1=\sqrt{3}.$

en $f'(\frac{4}{3}\pi)=2\sin(\frac{4}{3}\pi)\cdot\cos(\frac{4}{3}\pi)-\sqrt{3}\cdot\cos(\frac{4}{3}\pi)=2\cdot-\frac{1}{2}\sqrt{3}\cdot-\frac{1}{2}-\sqrt{3}\cdot-\frac{1}{2}=\frac{1}{2}\sqrt{3}+\frac{1}{2}\sqrt{3}=\sqrt{3}.$

62

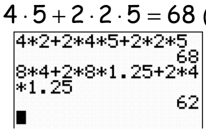
| Sinusoïde | toppen (binnen één periode) |
|---|---|
| $y=\sin(x)$ | $(\frac{1}{2}\pi,1)$ en $(\frac{3}{2}\pi,\ -1)$ |
| ⇩ verm. t.o.v. de $y$-as met $\frac{1}{c}$ | |
| $y=\sin(cx)$ | $(\frac{1}{c}\cdot\frac{1}{2}\pi,1)$ en $(\frac{1}{c}\cdot\frac{3}{2}\pi,\ -1)$ |
| ⇩ verm.t.o.v. de $x$-as met $b$ | |
| $y=b\sin(cx)$ | $(\frac{1}{2c}\pi,b)$ en $(\frac{3}{2c}\pi,\ -b)$ |
| ⇩ translatie $(d,a)$ | |
| $y=a+b\sin(c(x-d))$ | $(\frac{1}{2c}\pi+d,a+b)$ en $(\frac{3}{2c}\pi+d,a-b)$ |

| Sinusoïde | toppen (binnen één periode) |
|---|---|
| $y=\cos(x)$ | $(0,1)$ en $(\pi,\ -1)$ |
| ⇩ verm. t.o.v. de $y$-as met $\frac{1}{c}$ | |
| $y=\cos(cx)$ | $(0,1)$ en $(\frac{1}{c}\cdot\pi,\ -1)$ |
| ⇩ verm.t.o.v. de $x$-as met $b$ | |
| $y=b\cos(cx)$ | $(0,b)$ en $(\frac{1}{c}\pi,\ -b)$ |
| ⇩ translatie $(d,a)$ | |
| $y=a+b\,\mathrm{cosn}(c(x-d))$ | $(d,a+b)$ en $(\frac{1}{c}\pi+d,a-b)$ |

63 Geen opgave 63 te vinden.

64a $I=2x\cdot x\cdot h=40\Rightarrow h=\frac{40}{2x^2}=\frac{20}{x^2};$ $x=2$ (dm) $\Rightarrow h=\frac{20}{2^2}=\frac{20}{4}=5$ (dm) en $M=4\cdot 2+2\cdot 4\cdot 5+2\cdot 2\cdot 5=68$ (dm²).

64b $x=4$ (dm) $\Rightarrow h=\frac{20}{4^2}=\frac{20}{16}=\frac{5}{4}=1{,}25$ (dm) en $M=8\cdot 4+2\cdot 8\cdot 1{,}25+2\cdot 4\cdot 1{,}25=62$ (dm²).




64c $M=2\cdot 2x\cdot h+2\cdot x\cdot h+1\cdot 2x\cdot x=4xh+2xh+2x^2=6xh+2x^2$ (dm²).

65a $I = 2x \cdot x \cdot h = 72$ (dm$^3$) $\Rightarrow h = \frac{72}{2x^2} = \frac{36}{x^2}$ (dm).

$K = 0{,}4 \cdot 2x \cdot x + 0{,}2(2 \cdot 2x \cdot h + 2 \cdot x \cdot h) = 0{,}8x^2 + 1{,}2xh = 0{,}8x^2 + 1{,}2x \cdot \frac{36}{x^2} = 0{,}8x^2 + \frac{43{,}2}{x}$ (€).

```
1.2*36
                43.2
```

65b $K = 0{,}8x^2 + \frac{43{,}2}{x} = 0{,}8x^2 + 43{,}2x^{-1} \Rightarrow \frac{dK}{dx} = K' = 1{,}6x - 43{,}2x^{-2} = 1{,}6x - \frac{43{,}2}{x^2}$

$\frac{dK}{dx} = 0 \Rightarrow 1{,}6x - \frac{43{,}2}{x^2} = 0 \Rightarrow \frac{1{,}6x}{1} = \frac{43{,}2}{x^2} \Rightarrow 1{,}6x^3 = 43{,}2 \Rightarrow x^3 = \frac{43{,}2}{1{,}6} = 27 \Rightarrow x = \sqrt[3]{27} = 3$ (dm).

De materiaalkosten zijn minimaal (er is slechts 1 kandidaat) bij de afmetingen van 3 bij 6 bij 4 dm.

```
43.2/1.6
                  27
3×√27→X
                   3
2X
                   6
36/X²
                   4
```

66a $I = x \cdot x \cdot h = 16$ (dm$^3$) $\Rightarrow h = \frac{16}{x^2}$ ($h$ is de hoogte in dm).

$O = x \cdot x + 4 \cdot x \cdot h = x^2 + 4xh = x^2 + 4x \cdot \frac{16}{x^2} = x^2 + \frac{64}{x}$ (dm$^2$).

66b $O = x^2 + \frac{64}{x} = x^2 + 64x^{-1} \Rightarrow \frac{dO}{dx} = O' = 2x - 64x^{-2} = 2x - \frac{64}{x^2}$.

$\frac{dO}{dx} = 0 \Rightarrow 2x - \frac{64}{x^2} = 0 \Rightarrow \frac{2x}{1} = \frac{64}{x^2} \Rightarrow 2x^3 = 64 \Rightarrow x^3 = 32 \Rightarrow x = \sqrt[3]{32} \approx 3{,}17$ (dm).

```
3×√32→X
         3.174802104
16/X²
         1.587401052
```

De oppervlakte $O$ is minimaal (er is slechts 1 kandidaat) bij de afmetingen van 3,17 bij 3,17 bij 1,59 dm.

67 $O = x \cdot y = 75$ (m$^2$) $\Rightarrow y = \frac{75}{x}$.

$K = 10x + 20(x + 2y) = 30x + 40y = 30x + 40 \cdot \frac{75}{x} = 30x + \frac{3000}{x}$ (€).

```
40*75
                3000
```

$K = 30x + \frac{3000}{x} = 30x + 3000x^{-1} \Rightarrow \frac{dK}{dx} = K' = 30 - 3000x^{-2} = 30 - \frac{3000}{x^2}$.

$\frac{dK}{dx} = 0 \Rightarrow 30 - \frac{3000}{x^2} = 0 \Rightarrow \frac{30}{1} = \frac{3000}{x^2} \Rightarrow 30x^2 = 3000 \Rightarrow x^2 = 100$ (met $x > 0$) $\Rightarrow x = \sqrt{100} = 10$ (m).

De kosten $K$ zijn minimaal (er is slechts 1 kandidaat) bij de afmetingen 10 (voor de vierde zijde) bij 7,5 m.

```
√(100)→X
                  10
75/X
                 7.5
```

68a $O = x \cdot y = 1200$ (m$^2$) $\Rightarrow y = \frac{1200}{x}$.

$K = 60y + 15(x + y) = 15x + 75y = 15x + 75 \cdot \frac{1200}{x} = 15x + \frac{90000}{x}$ (€).

```
75*1200
               90000
Ans/15
                6000
√(6000)→X
         77.45966692
```

68b $K = 15x + \frac{90000}{x} = 15x + 90\,000x^{-1} \Rightarrow \frac{dK}{dx} = K' = 15 - 90\,000x^{-2} = 15 - \frac{90000}{x^2}$.

$\frac{dK}{dx} = 0 \Rightarrow 15 - \frac{90000}{x^2} = 0 \Rightarrow \frac{15}{1} = \frac{90000}{x^2} \Rightarrow 15x^2 = 90\,000 \Rightarrow x^2 = 6\,000$ (met $x > 0$) $\Rightarrow x = \sqrt{6\,000} \approx 77{,}5$ (m).

De minimale kosten (1 kandidaat) zijn € 2 323,79
bij de afmetingen 77,5 (langs de beek) bij 15,5 m.

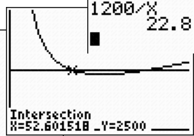
```
15X+90000/X
         2323.790008
1200/X
         15.49193338
```

68c $K = 15x + \frac{90000}{x} = 2\,500$ intersect geeft $x \approx 52{,}6$ (minder lang).

De afmetingen zijn 52,6 bij 22,8 meter.

```
Plot1 Plot2 Plot3
\Y1=15X+90000/X
\Y2=2500
\Y3=
\Y4=
```

```
WINDOW
 Xmin=0
 Xmax=150
 Xscl=0
 Ymin=0
 Ymax=5000
 Yscl=0
 Xres=1
```




```
X
         52.60151848
1200/X
         22.81302964
```

69a $O = 2 \cdot \pi r^2$ (bodem en deksel) $+ 2\pi r \cdot h$ (mantel) $= 2\pi r^2 + 2\pi rh$ (cm$^2$).

69b $I = \pi r^2 \cdot h = 1000$ (cm$^3$) $\Rightarrow h = \frac{1000}{\pi r^2}$ (cm).

69c $O = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2} = 2\pi r^2 + \frac{2000}{r}$ (cm$^2$).

69d $O = 2\pi r^2 + \frac{2000}{r} = 2\pi r^2 + 2000r^{-1} \Rightarrow \frac{dO}{dr} = O' = 4\pi r - 2000r^{-2} = 4\pi r - \frac{2000}{r^2}$.

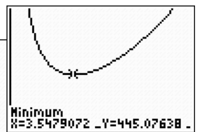
$\frac{dO}{dr} = 0 \Rightarrow 4\pi r - \frac{2000}{r^2} = 0 \Rightarrow \frac{4\pi r}{1} = \frac{2000}{r^2} \Rightarrow 4\pi r^3 = 2000 \Rightarrow r^3 = \frac{500}{\pi} \Rightarrow r = \sqrt[3]{\frac{500}{\pi}} \approx 5{,}4$ (cm).

```
3×√(500/π)→X
         5.419260701
1000/(πX²)
          10.8385214
```

De hoeveelheid materiaal is minimaal (er is slechts 1 kandidaat) bij $r = \sqrt[3]{\frac{500}{\pi}} \approx 5{,}4$ cm en $h = \frac{1000}{\pi r^2} \approx 10{,}8$ cm.

```
Plot1 Plot2 Plot3
\Y1=3πX²+4πX+100
0/X
\Y2=
```

```
WINDOW
 Xmin=0
 Xmax=10
 Xscl=0
 Ymin=0
 Ymax=1000
 Yscl=0
 Xres=1
```




70a $I = \pi r^2 \cdot h = 500$ (cm$^3$) $\Rightarrow h = \frac{500}{\pi r^2}$ (cm).

$K = 1 \cdot \pi r^2$ (bodem) $+ 1 \cdot 2\pi rh$ (mantel) $+ 2 \cdot \pi r^2$ (deksel) $+ 2 \cdot 2\pi r \cdot 1$ (rand met hoogte 1 cm)
$= 3\pi r^2 + 4\pi r + 2\pi rh = 3\pi r^2 + 4\pi r + 2\pi r \cdot \frac{500}{\pi r^2} = 3\pi r^2 + 4\pi r + \frac{1000}{r}$.

70b De materiaalkosten zijn minimaal (optie minimum is toegestaan) bij $r \approx 3{,}5$ cm en $h \approx 12{,}6$ cm.
(oplossen met de afgeleide geeft een derdegraadsvergelijking die niet algebraïsch kan worden opgelost,
deze vergelijking kan vervolgens dan met intersect grafisch-numeriek worden opgelost)

```
X
         3.547907246
500/(πX²)
         12.64374175
```

71a $I = \pi r^2 \cdot h \Rightarrow h = \frac{I}{\pi r^2}$. (hierin is $I$ een bepaalde inhoud dus voor te stellen als een of ander vast getal)

71b $O = 2 \cdot \pi r^2$ (bodem en deksel) $+ 2\pi r \cdot h$ (mantel) $= 2\pi r^2 + 2\pi r \cdot \frac{I}{\pi r^2} = \frac{2I}{r} + 2\pi r^2$.

71c $O = \frac{2I}{r} + 2\pi r^2 = 2I \cdot r^{-1} + 2\pi r^2 \Rightarrow \frac{\mathrm{d}O}{\mathrm{d}r} = O' = -2I \cdot r^{-2} + 4\pi r = 4\pi r - \frac{2I}{r^2}$.

$\frac{\mathrm{d}O}{\mathrm{d}r} = 0 \Rightarrow 4\pi r - \frac{2I}{r^2} = 0 \Rightarrow \frac{4\pi r}{1} = \frac{2I}{r^2} \Rightarrow 4\pi r^3 = 2I \Rightarrow r^3 = \frac{I}{2\pi} \Rightarrow r = \sqrt[3]{\frac{I}{2\pi}}$.

Dus $O$ is minimaal (er is slechts 1 kandidaat) bij $r = \sqrt[3]{\frac{I}{2\pi}}$.

71d $r = \sqrt[3]{\frac{I}{2\pi}} = \sqrt[3]{\frac{\pi r^2 h}{2\pi}}$ (alles tot de derde macht nemen)

$\frac{r^3}{1} = \frac{\pi r^2 h}{2\pi}$ (kruiselings vermenigvuldigen)

$2\pi r^3 = \pi r^2 h$ (links en rechts delen door $\pi r^2$)

$2r = h$.

72a $AB + AC + BC = 12$

$x + AC + AC = 12$

$2AC = 12 - x$

$AC = \frac{12 - x}{2}$

$AC = \frac{12}{2} - \frac{x}{2}$

$AC = 6 - \frac{1}{2}x$.

72b $CD^2 = AC^2 - AD^2$

$= (6 - \frac{1}{2}x)^2 - (\frac{1}{2}x)^2 = (6 - \frac{1}{2}x)(6 - \frac{1}{2}x) - (\frac{1}{2}x)^2$

$= 36 - 3x - 3x + \frac{1}{4}x^2 - \frac{1}{4}x^2 = 36 - 6x$.

$CD = \sqrt{36 - 6x}$.

72c $O(ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2}x \cdot \sqrt{36 - 6x}$.

73a $K = 12 \cdot (200 - x) + 10 \cdot \sqrt{x^2 + 3600} \cdot 2 = 2\,400 - 12x + 20 \cdot \sqrt{x^2 + 3600}$ (€).

73b $K = 2\,400 - 12x + 20 \cdot \sqrt{x^2 + 3600} \Rightarrow \frac{\mathrm{d}K}{\mathrm{d}x} = K' = -12 + 20 \cdot \frac{1}{2 \cdot \sqrt{x^2 + 3600}} \cdot 2x = -12 + \frac{20x}{\sqrt{x^2 + 3600}}$. $[\sqrt{x}]' = \frac{1}{2 \cdot \sqrt{x}}$.

$\frac{\mathrm{d}K}{\mathrm{d}x} = 0 \Rightarrow -12 + \frac{20x}{\sqrt{x^2 + 3600}} = 0 \Rightarrow \frac{20x}{\sqrt{x^2 + 3600}} = \frac{12}{1} \Rightarrow 20x = 12 \cdot \sqrt{x^2 + 3600} \Rightarrow 400x^2 = 144(x^2 + 3600)$

$400x^2 = 144x^2 + 144 \cdot 3600 \Rightarrow 256x^2 = 518\,400 \Rightarrow x^2 = 2\,025$ (met $x > 0$) $\Rightarrow x = 45$ (m).

De minimale kosten (er is slechts 1 kandidaat) zijn € 3360 bij $x = 45$ m.

```
2400-12X+20√(X²+
3600)
            3360
```

```
144*3600
          518400
Ans/256
            2025
√(2025)→X
              45
```

74a $K = a \cdot (200 - x) + 10 \cdot \sqrt{x^2 + 3600} \cdot 2 = 200a - ax + 20 \cdot \sqrt{x^2 + 3600}$ (€). (hierbij is $a$ een constante!!!)

$K = 200a - ax + 20 \cdot \sqrt{x^2 + 3600} \Rightarrow \frac{\mathrm{d}K}{\mathrm{d}x} = K' = -a + 20 \cdot \frac{1}{2 \cdot \sqrt{x^2 + 3600}} \cdot 2x = -a + \frac{20x}{\sqrt{x^2 + 3600}}$.

74b $\left[\frac{\mathrm{d}K}{\mathrm{d}x}\right]_{x = 200 - AP} = 0 \Rightarrow \left[\frac{\mathrm{d}K}{\mathrm{d}x}\right]_{x = 200 - 140 = 60} = 0 \Rightarrow -a + \frac{20 \cdot 60}{\sqrt{60^2 + 3600}} = 0 \Rightarrow \frac{1200}{\sqrt{7200}} = a \approx 14{,}14$ (€).

```
1200/√(7200)
     14.14213562
```

75a $AB' = \sqrt{500^2 + 200^2} = \sqrt{250\,000 + 40\,000} = \sqrt{290\,000}$ (m).

De totale kosten via het traject $AB'B$ zijn $100 \cdot \sqrt{290\,000} + 150 \cdot 100 \approx 68\,852$ (€).

```
500²+200²
          290000
100*√(290000)+15
0*100
     68851.64807
```

75b $AB = \sqrt{500^2 + 300^2} = \sqrt{340\,000}$ (m). Verder is $BC = \frac{100}{300} \cdot \sqrt{340\,000}$ (m) en $AC = \frac{200}{300} \cdot \sqrt{340\,000}$ (m).

Een kabel in een rechte lijn van $A$ naar $B$ kost $100 \cdot \frac{2}{3} \cdot \sqrt{340\,000} + 150 \cdot \frac{1}{3} \cdot \sqrt{340\,000} \approx 68\,852$ (€).

```
500²+300²
          340000
(200/3+150/3)*√(
340000)
     68027.77211
```

75c $AP = \sqrt{x^2 + 200^2} = \sqrt{x^2 + 40\,000}$ (m) en $PB = \sqrt{(500 - x)^2 + 100^2} = \sqrt{(500 - x)(500 - x) + 100^2}$

$= \sqrt{250\,000 - 500x - 500x + x^2 + 10\,000} = \sqrt{x^2 - 1000x + 260\,000}$ (m).

De kosten voor een kabel via punt $P$ zijn $K = 100 \cdot \sqrt{x^2 + 40\,000} + a \cdot \sqrt{x^2 - 1000x + 260\,000}$ (€).

$K = 100 \cdot \sqrt{x^2 + 40\,000} + a \cdot \sqrt{x^2 - 1000x + 260\,000}$ heeft als afgeleide functie

$\frac{\mathrm{d}K}{\mathrm{d}x} = K' = 100 \cdot \frac{1}{2 \cdot \sqrt{x^2 + 40000}} \cdot 2x + a \cdot \frac{1}{2 \cdot \sqrt{x^2 - 1000x + 260000}} \cdot (2x - 1000) = \frac{100x}{\sqrt{x^2 + 40000}} + \frac{a(x - 500)}{\sqrt{x^2 - 1000x + 260000}}$.

75d $\left[\frac{\mathrm{d}K}{\mathrm{d}x}\right]_{x=400} = 0 \Rightarrow \frac{100 \cdot 400}{\sqrt{400^2 + 40000}} + \frac{a(400 - 500)}{\sqrt{400^2 - 1000 \cdot 400 + 260000}} = 0$

$\frac{-100a}{\sqrt{400^2 - 1000 \cdot 400 + 260000}} = -\frac{100 \cdot 400}{\sqrt{400^2 + 40000}}$

$a = \frac{100 \cdot 400}{\sqrt{400^2 + 40000}} \cdot \frac{\sqrt{400^2 - 1000 \cdot 400 + 260000}}{100} \approx 126{,}49$ (€/m).

```
100*400/√(400²+4
0000)*√(400²-100
0*400+260000)/10
0
     126.4911064
```

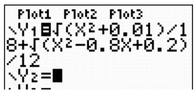
76a $AP = \sqrt{x^2 + 0{,}1^2} = \sqrt{x^2 + 0{,}01}$ (km) en $BP = \sqrt{(0{,}4 - x)^2 + 0{,}2^2} = \sqrt{(0{,}4 - x)(0{,}4 - x) + 0{,}2^2}$

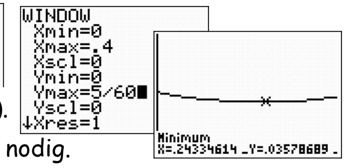
$= \sqrt{0{,}16 - 0{,}4x - 0{,}4x + x^2 + 0{,}04} = \sqrt{x^2 - 0{,}8x + 0{,}20}$ (km).

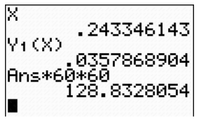
snelheid $= \frac{\text{afstand}}{\text{tijd}} \Rightarrow$ tijd $= \frac{\text{afstand}}{\text{snelheid}}$

De totale tijd $t$ is gelijk aan $t = \frac{\sqrt{x^2 + 0{,}01}}{18} + \frac{\sqrt{x^2 - 0{,}8x + 0{,}2}}{12} = \frac{1}{18} \cdot \sqrt{x^2 + 0{,}01} + \frac{1}{12} \cdot \sqrt{x^2 - 0{,}8x + 0{,}2}$ (uur).

76b Maak een schets van jouw plot.
Zie een voorbeeld hiernaast.
Vermeld het WINDOW-scherm.




76c De optie minimum geeft als top het punt $(x, t) = (0{,}243...; 0{,}035...)$.

76d Frits heeft minimaal 0,035... uur $= 0{,}035... \cdot 60 \cdot 60 \approx 129$ seconden nodig.

77a Van $S$(TART) naar aankomst $L$(and) is $SL = \sqrt{x^2 + 2^2} = \sqrt{x^2 + 4}$ (km) en van $L$ naar $F$(INISH) is $LF = 10 - x$ (km).

De totale tijd $t$ is gelijk aan $t = \frac{\sqrt{x^2 + 4}}{4} + \frac{10 - x}{12} = \frac{1}{4} \cdot \sqrt{x^2 + 4} + \frac{1}{12} \cdot (10 - x)$ (uur).

77b $t = \frac{1}{4} \cdot \sqrt{x^2 + 4} + \frac{1}{12} \cdot (10 - x) \Rightarrow \frac{dt}{dx} = t' = \frac{1}{4} \cdot \frac{1}{2 \cdot \sqrt{x^2 + 4}} \cdot 2x + \frac{1}{12} \cdot -1 = \frac{x}{4 \cdot \sqrt{x^2 + 4}} - \frac{1}{12}$.

$\frac{dt}{dx} = 0 \Rightarrow \frac{x}{4 \cdot \sqrt{x^2 + 4}} = \frac{1}{12} \Rightarrow 12x = 4 \cdot \sqrt{x^2 + 4} \Rightarrow 3x = \sqrt{x^2 + 4} \Rightarrow 9x^2 = x^2 + 4 \Rightarrow 8x^2 = 4 \Rightarrow x^2 = \frac{1}{2}$ $(x > 0) \Rightarrow x = \sqrt{\frac{1}{2}}$.

Dus $t$ is minimaal (er is slechts 1 kandidaat) voor $x = \sqrt{\frac{1}{2}}$ $(\approx 0{,}707)$ km.

78a $AB + BC = x + BC = 60 \Rightarrow BC = 60 - x$. (nu de stelling van Pythagoras in $\Delta ABC$)

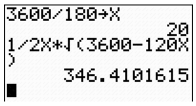
$AC = \sqrt{BC^2 - AB^2} = \sqrt{(60 - x)^2 - x^2} = \sqrt{(60 - x)(60 - x) - x^2} = \sqrt{3600 - 60x - 60x + x^2 - x^2} = \sqrt{3600 - 120x}$.

$O_{ABC} = \frac{1}{2} \cdot AB \cdot AC = \frac{1}{2} x \cdot \sqrt{3600 - 120x}$.

78b $O = \frac{1}{2} x \cdot \sqrt{3600 - 120x} \Rightarrow \frac{dO}{dx} = O' = \frac{1}{2} \cdot \sqrt{3600 - 120x} + \frac{1}{2} x \cdot \frac{1}{2 \cdot \sqrt{3600 - 120x}} \cdot -120 = \frac{\sqrt{3600 - 120x}}{2} - \frac{30x}{\sqrt{3600 - 120x}}$.

$\frac{dO}{dx} = 0 \Rightarrow \frac{\sqrt{3600 - 120x}}{2} = \frac{30x}{\sqrt{3600 - 120x}} \Rightarrow 60x = 3600 - 120x \Rightarrow 180x = 3600 \Rightarrow x = \frac{3600}{180} = 20$.

De maximale (er is slechts 1 kandidaat) oppervlakte $O(20) \approx 346{,}41$.




79a $AP + PD = x + PD = 20 \Rightarrow PD = 20 - x$. (nu de stelling van Pythagoras in $\Delta ADP$)

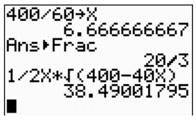
$AD = \sqrt{PD^2 - AP^2} = \sqrt{(20 - x)^2 - x^2} = \sqrt{(20 - x)(20 - x) - x^2} = \sqrt{400 - 20x - 20x + x^2 - x^2} = \sqrt{400 - 40x}$.

$O_{ABC} = \frac{1}{2} \cdot AD \cdot AP = \frac{1}{2} \cdot \sqrt{400 - 40x} \cdot x = \frac{1}{2} x \cdot \sqrt{400 - 40x}$.

79b $O = \frac{1}{2} x \cdot \sqrt{400 - 40x} \Rightarrow \frac{dO}{dx} = O' = \frac{1}{2} \cdot \sqrt{400 - 40x} + \frac{1}{2} x \cdot \frac{1}{2 \cdot \sqrt{400 - 40x}} \cdot -40 = \frac{\sqrt{400 - 40x}}{2} - \frac{10x}{\sqrt{400 - 40x}}$.

$\frac{dO}{dx} = 0 \Rightarrow \frac{\sqrt{400 - 40x}}{2} = \frac{10x}{\sqrt{400 - 40x}} \Rightarrow 20x = 400 - 40x \Rightarrow 60x = 400 \Rightarrow x = \frac{400}{60} = \frac{40}{6} = \frac{20}{3}$ (cm).

De maximale (er is slechts 1 kandidaat) oppervlakte $O(\frac{20}{3}) \approx 38{,}49$ cm$^2$.



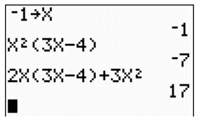
**Diagnostische toets**

D1a $f(x)=x^3(2x+1) \Rightarrow f'(x)=3x^2(2x+1)+x^3\cdot 2 = 3x^2(2x+1)+2x^3.$

D1b $g(x)=(x^2-2)(3x^2+4) \Rightarrow g'(x)=2x(3x^2+4)+(x^2-2)\cdot 6x.$

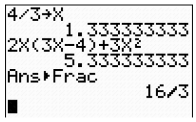
D1c $h(x)=(x^2-4)^2=(x^2-4)(x^2-4) \Rightarrow h'(x)=2x(x^2-4)+(x^2-4)\cdot 2x=4x(x^2-4).$

D2a $f(x)=x^2(3x-4) \Rightarrow f'(x)=2x(3x-4)+x^2\cdot 3=2x(3x-4)+3x^2.$
$y_A=f(-1)=-7$ en $rc_{raaklijn}=f'(-1)=17.$
$k\!: y=17x+b$ door $A(-1,\,-7) \Rightarrow 17\cdot -1+b=-7 \Rightarrow b=-7+17=10.$ Dus $k\!: y=17x+10.$



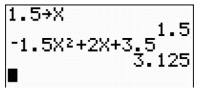

D2b $f'(-1)=2(3-4)+3\cdot 1=-2+3\neq 0.$ Dus de grafiek van $f$ heeft geen top voor $x=1.$

D2c Snijden met de $x$-as $(y=0) \Rightarrow f(x)=0 \Rightarrow x^2(3x-4)=0 \Rightarrow x=0 \vee 3x=4.$ Dus $x_B=\frac{4}{3}.$
$y_B=f(\frac{4}{3})=0$ $(y=0)$ en $rc_{raaklijn}=f'(\frac{4}{3})=\frac{16}{3}.$
$k\!: y=\frac{16}{3}x+b$ door $B(\frac{4}{3},\,0) \Rightarrow \frac{16}{3}\cdot\frac{4}{3}+b=0 \Rightarrow b=0-\frac{16}{3}\cdot\frac{4}{3}=-\frac{64}{9}.$ Dus $k\!: y=\frac{16}{3}x-\frac{64}{9}.$




D3a $f(x)=-x^2+3x+4=0 \Rightarrow x^2-3x-4=0 \Rightarrow (x-4)(x+1)=0 \Rightarrow x=4 \vee x_A=-1.$
Nu is: $AB=x_B-x_A=p--1=p+1$ en $BC=y_C=-p^2+3p+4.$
$O(\Delta ABC)=\frac{1}{2}\cdot AB\cdot BC=\frac{1}{2}\cdot(p+1)\cdot(-p^2+3p+4)=(\frac{1}{2}p+\frac{1}{2})(-p^2+3p+4).$

D3b $O(p)=(\frac{1}{2}p+\frac{1}{2})(-p^2+3p+4) \Rightarrow O'(p)=\frac{1}{2}(-p^2+3p+4)+(\frac{1}{2}p+\frac{1}{2})\cdot(-2p+3)$
$=-\frac{1}{2}p^2+1\frac{1}{2}p+2-p^2+1\frac{1}{2}p-p+1\frac{1}{2}=-1\frac{1}{2}p^2+2p+3\frac{1}{2}.$
$x_{top}=\frac{-1+4}{2}=\frac{3}{2}$ en $O'(\frac{3}{2})=3{,}125\neq 0.$ Dus $O$ is niet maximaal voor $p=\frac{3}{2}.$




D3c $O'(p)=0 \Rightarrow -1\frac{1}{2}p^2+2p+3\frac{1}{2}=0$ (keer $-2$)
$3p^2-4p-7=0$ met $D=(-4)^2-4\cdot 3\cdot -7=16+84=100$
$p=\frac{4-10}{2\cdot 3}=\frac{-6}{6}=-1$ ( $\leq -1$ voldoet niet) $\vee\ p=\frac{4+10}{6}=\frac{14}{6}=\frac{7}{3}.$ Dus $O$ maximaal voor $p=\frac{7}{3}=2\frac{1}{3}.$

D4a $f(x)=\frac{3}{x^4}=3\cdot\frac{1}{x^4}=3x^{-4} \Rightarrow f'(x)=-12x^{-5}=-12\cdot\frac{1}{x^5}=-\frac{12}{x^5}.$

D4b $g(x)=4x^3-\frac{3}{x^3}=4x^3-3x^{-3} \Rightarrow g'(x)=12x^2+9x^{-4}=12x^2+\frac{9}{x^4}.$

D4c $h(x)=\frac{2x^3-3}{x^3}=\frac{2x^3}{x^3}-\frac{3}{x^3}=2-3x^{-3} \Rightarrow h'(x)=9x^{-4}=\frac{9}{x^4}.$

D4d $k(x)=\frac{6-x^2}{x}=\frac{6}{x}-\frac{x^2}{x}=6x^{-1}-x \Rightarrow k'(x)=-6x^{-2}-1=-\frac{6}{x^2}-1.$

D4e $l(x)=\frac{1}{3x^6}=\frac{1}{3}\cdot\frac{1}{x^6}=\frac{1}{3}x^{-6} \Rightarrow l'(x)=-2x^{-7}=-\frac{2}{x^7}.$

D4f $m(x)=\frac{x^2+2x+1}{x}=\frac{x^2}{x}+\frac{2x}{x}+\frac{1}{x}=x+2+x^{-1} \Rightarrow m'(x)=1-x^{-2}=1-\frac{1}{x^2}.$

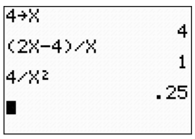
D5a
$\frac{x+1}{x}=\frac{2}{3}$
$3(x+1)=2x$
$3x+3=2x$
$x=-3.$

D5b
$\frac{6}{x^2}=\frac{2x}{9}$
$2x^3=54$
$x^3=27=3^3$
$x=3.$

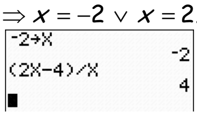
D5c
$\frac{-3}{x^2}=-\frac{1}{3}$
$\frac{3}{x^2}=\frac{1}{3}$
$x^2=9$
$x=3 \vee x=-3.$

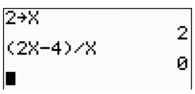
D5d
$\frac{x-3}{x+2}=\frac{x+1}{x+26}$
$(x-3)(x+26)=(x+1)(x+2)$
$x^2+23x-78=x^2+3x+2$
$20x=80$
$x=4.$

D6a $f(x)=\frac{2x-4}{x}=\frac{2x}{x}-\frac{4}{x}=2-4x^{-1} \Rightarrow f'(x)=4x^{-2}=\frac{4}{x^2}.$
$y_A=f(4)=1$ en $rc_{raaklijn}=f'(4)=\frac{1}{4}.$
$k\!: y=\frac{1}{4}x+b$ door $A(4,1) \Rightarrow \frac{1}{4}\cdot 4+b=1 \Rightarrow b=1-1=0.$ Dus $k\!: y=\frac{1}{4}x.$




D6b $rc_{raaklijn}=f'(x)=1 \Rightarrow \frac{4}{x^2}=1 \Rightarrow x^2=4 \Rightarrow x=-2 \vee x=2.$
$y_B=f(-2)=4$ en $y_C=f(2)=0.$
De raakpunten zijn $B(-2,\,4)$ en $C(2,\,0).$



D7a $f(x)=2x^2\cdot\sqrt{x}=2x^2\cdot x^{\frac12}=2x^{2\frac12}\Rightarrow f'(x)=5x^{1\frac12}=5x^1\cdot x^{\frac12}=5x\cdot\sqrt{x}$.

D7b $g(x)=(x+4)\cdot\sqrt{x}=x\cdot x^{\frac12}+4x^{\frac12}=x^{1\frac12}+4x^{\frac12}\Rightarrow g'(x)=1\frac12 x^{\frac12}+2x^{-\frac12}=1\frac12\cdot\sqrt{x}+\frac{2}{x^{\frac12}}=1\frac12\cdot\sqrt{x}+\frac{2}{\sqrt{x}}$.

Of $g(x)=(x+4)\cdot\sqrt{x}$ (productregel) $\Rightarrow g'(x)=1\cdot\sqrt{x}+(x+4)\cdot[x^{\frac12}]'=1\cdot\sqrt{x}+(x+4)\cdot\frac12 x^{-\frac12}=\sqrt{x}+\frac{x+4}{2\cdot\sqrt{x}}$.

D7c $h(x)=\frac{x+4}{\sqrt[3]{x}}=\frac{x+4}{x^{\frac13}}=\frac{x}{x^{\frac13}}+\frac{4}{x^{\frac13}}=x^{\frac23}+4x^{-\frac13}\Rightarrow h'(x)=\frac23 x^{-\frac13}-\frac43 x^{-1\frac13}=\frac{2}{3x^{\frac13}}-\frac{4}{3x\cdot x^{\frac13}}=\frac{2}{3\cdot\sqrt[3]{x}}-\frac{4}{3x\cdot\sqrt[3]{x}}$.

D7d $k(x)=(x^3+x)(1-\sqrt{x})\Rightarrow k'(x)=(3x^2+1)(1-\sqrt{x})+(x^3+x)\cdot-\frac12 x^{-\frac12}=(3x^2+1)(1-\sqrt{x})-\frac{x^3+x}{2\cdot\sqrt{x}}$.

D8a $f(x)=3(x^2+4x)^4\Rightarrow f'(x)=12(x^2+4x)^3\cdot(2x+4)$.

D8b $g(x)=\sqrt{x^2+2}\Rightarrow g'(x)=\frac{1}{2\cdot\sqrt{x^2+2}}\cdot 2x=\frac{x}{\sqrt{x^2+2}}$. Gebruik: $[\sqrt{x}]'=\frac{1}{2\cdot\sqrt{x}}$.

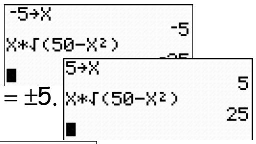
D8c $h(x)=x^2(2x-1)^4\Rightarrow h'(x)=2x(2x-1)^4+x^2\cdot4(2x-1)^3\cdot2=2x(2x-1)^4+8x^2(2x-1)^3$.

D8d $k(x)=x\cdot\sqrt{2-x}\Rightarrow k'(x)=1\cdot\sqrt{2-x}+x\cdot\frac{1}{2\cdot\sqrt{2-x}}\cdot-1=\sqrt{2-x}-\frac{x}{2\cdot\sqrt{2-x}}$.

D9a $f(x)=x\cdot\sqrt{50-x^2}\Rightarrow f'(x)=1\cdot\sqrt{50-x^2}+x\cdot\frac{1}{2\cdot\sqrt{50-x^2}}\cdot-2x=\sqrt{50-x^2}-\frac{x^2}{\sqrt{50-x^2}}$.

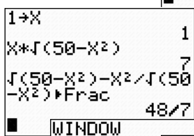
$f'(x)=0\Rightarrow\sqrt{50-x^2}-\frac{x^2}{\sqrt{50-x^2}}=0\Rightarrow\frac{\sqrt{50-x^2}}{1}=\frac{x^2}{\sqrt{50-x^2}}\Rightarrow 1\cdot x^2=50-x^2\Rightarrow x^2=50\Rightarrow x=\pm5$.

$f(-5)=-5\cdot\sqrt{25}=-25$ en $f(5)=5\cdot\sqrt{25}=25$. De toppen zijn $(-5,\ -25)$ en $(5,25)$.

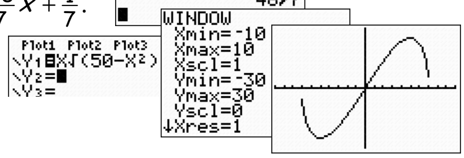



D9b $y_A=f(1)=1\cdot\sqrt{49}=7$ en $rc_{raaklijn}=f'(1)=\sqrt{49}-\frac{1}{\sqrt{49}}=7-\frac17=\frac{48}{7}$.

$k$: $y=\frac{48}{7}x+b$ door $A(1,7)\Rightarrow\frac{48}{7}\cdot1+b=7\Rightarrow b=7-\frac{48}{7}=\frac17$. Dus $k$: $y=\frac{48}{7}x+\frac17$.




D9c $D_f=\left[-\sqrt{50},\sqrt{50}\right]$ (BV: $50-x^2\ge0\Rightarrow-x^2\ge-50\Rightarrow x^2\le50\Rightarrow-\sqrt{50}\le x\le\sqrt{50}$).
$B_f=\left[-25,25\right]$ (zie D9a en een plot).




D10a $\sin(\alpha)=-\frac12\sqrt3$ $(0\le\alpha\le2\pi)\Rightarrow\alpha=1\frac13\pi\vee\alpha=1\frac23\pi$.

D10b $\cos(\alpha)=-\frac12\sqrt3$ $(0\le\alpha\le2\pi)\Rightarrow\alpha=\frac34\pi\vee\alpha=1\frac14\pi$.

D10c $\sin(\alpha)=\frac12$ $(0\le\alpha\le2\pi)\Rightarrow\alpha=\frac16\pi\vee\alpha=\frac56\pi$.

D10d $\cos(\alpha)=\frac12$ $(0\le\alpha\le2\pi)\Rightarrow\alpha=\frac13\pi\vee\alpha=1\frac23\pi$.

D11a $4\sin(2x+\frac12\pi)=2\sqrt2$
$\sin(2x+\frac12\pi)=\frac12\sqrt2$
$2x+\frac12\pi=\frac14\pi+k\cdot2\pi\vee2x+\frac12\pi=\frac34\pi+k\cdot2\pi$
$2x=-\frac14\pi+k\cdot2\pi\vee2x=\frac14\pi+k\cdot2\pi$
$x=-\frac18\pi+k\cdot\pi\vee x=\frac18\pi+k\cdot\pi$. (met $x$ op $[0,2\pi]$)
$x=\frac78\pi\vee x=1\frac78\pi\vee x=\frac18\pi\vee x=1\frac18\pi$.

D11b $\cos(x-\frac16\pi)=-\frac12\sqrt3$
$x-\frac16\pi=\frac56\pi+k\cdot2\pi\vee x-\frac16\pi=-\frac56\pi+k\cdot2\pi$
$x=\pi+k\cdot2\pi\vee x=-\frac23\pi+k\cdot2\pi$. (met $x$ op $[0,2\pi]$)
$x=\pi\vee x=\frac43\pi$.

D11c $\cos(1\frac12x)=-1$
$\frac32x=\pi+k\cdot2\pi$ (keer $\frac23$)
$x=\frac23\pi+k\cdot\frac43\pi$. (met $x$ op $[0,2\pi]$)
$x=\frac23\pi\vee x=2\pi$.

D11d $2\sin(3x+\frac12\pi)=-\sqrt3$
$\sin(3x+\frac12\pi)=-\frac12\sqrt3$
$3x+\frac12\pi=\frac43\pi+k\cdot2\pi\vee3x+\frac12\pi=-\frac13\pi+k\cdot2\pi$
$3x=\frac56\pi+k\cdot2\pi\vee3x=-\frac56\pi+k\cdot2\pi$
$x=\frac{5}{18}\pi+k\cdot\frac23\pi\vee x=-\frac{5}{18}\pi+k\cdot\frac23\pi$. (met $x$ op $[0,2\pi]$)
$x=\frac{5}{18}\pi\vee x=\frac{17}{18}\pi\vee x=\frac{29}{18}\pi\vee x=\frac{7}{18}\pi\vee x=\frac{19}{18}\pi\vee x=\frac{31}{18}\pi$.

D12a $f(x)=x^2+2\cos(x)\Rightarrow f'(x)=2x+2\sin(x)$.

D12b $g(x)=2x^2\cdot\cos(x)\Rightarrow g'(x)=4x\cdot\cos(x)+2x^2\cdot-\sin(x)=4x\cos(x)-2x^2\cdot\sin(x)$.

D12c $h(x)=\cos(x^2)\Rightarrow h'(x)=-\sin(x^2)\cdot2x=-2x\sin(x^2)$.

D12d $j(x)=\cos^2(2x)=\cos(2x)\cdot\cos(2x)\Rightarrow j'(x)=-\sin(2x)\cdot2\cdot\cos(2x)+\cos(2x)\cdot-\sin(2x)\cdot2=-4\sin(2x)\cdot\cos(2x)$.

Of $j(x)=\cos^2(2x)=\left(\cos(2x)\right)^2\Rightarrow j'(x)=2\cos(2x)\cdot-\sin(2x)\cdot2=-4\sin(2x)\cdot\cos(2x)$

D13a $h(t) = 0{,}1\sin(100\pi t) \Rightarrow h'(t) = 0{,}1\cos(100\pi t) \cdot 100\pi = 10\pi\cos(100\pi t)$.

D13b $u(t) = 4\sin(80\pi t) + 5\cos(81\pi t) \Rightarrow u'(t) = 4\cos(80\pi t) \cdot 80\pi + 5 \cdot -\sin(81\pi t) \cdot 81\pi = 320\pi\cos(80\pi t) - 405\pi\sin(81\pi t)$.

D13c $f(x) = 5x \cdot \sin(3x) \Rightarrow f'(x) = 5 \cdot \sin(3x) + 5x \cdot \cos(3x) \cdot 3 = 5\,\text{sins}(3x) + 15x\cos(3x)$.

D13d $g(x) = \sqrt{2 + \cos(x)} \Rightarrow g'(x) = \frac{1}{2 \cdot \sqrt{2+\cos(x)}} \cdot -\sin(x) = \frac{-\sin(x)}{2 \cdot \sqrt{2+\cos(x)}}$. Gebruik: $[\sqrt{x}]' = \frac{1}{2 \cdot \sqrt{x}}$.

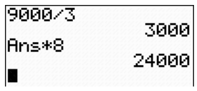
D14a $f(x) = x\cos^2(x) = x \cdot \left(\cos(x)\right)^2 \Rightarrow f'(x) = 1 \cdot \cos^2(x) + x \cdot 2\cos(x) \cdot -\sin(x) = \cos^2(x) - 2x\sin(x) \cdot \cos(x)$.

D14b $y_A = f(\pi) = \pi\cos^2(\pi) = \pi \cdot (-1)^2 = \pi$ en $rc_{raaklijn} = f'(\pi) = \cos^2(\pi) - 2\pi\sin(\pi) \cdot \cos(\pi) = (-1)^2 - 2\pi \cdot 0 = 1$.
$k$: $y = x + b$ door $A(\pi, \pi) \Rightarrow 1 \cdot \pi + b = \pi \Rightarrow b = \pi - \pi = 0$. Dus $k$: $y = x$.

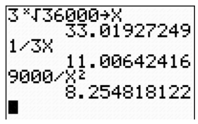
D15a $AB + AC = x + AC = 20 \Rightarrow AC = 20 - x$. (nu de stelling van Pythagoras in $\Delta ABC$)
$BC = \sqrt{AC^2 + AC^2} = \sqrt{x^2 + (20-x)^2} = \sqrt{x^2 + (20-x)(20-x)} = \sqrt{x^2 + 400 - 20x - 20x + x^2} = \sqrt{2x^2 - 40x + 400}$.
$O_{ABEDC} = O_{ABC} + O_{BEDC} = \frac{1}{2} \cdot AB \cdot AC + BC^2$
$= \frac{1}{2} \cdot x \cdot (20 - x) + 2x^2 - 40x + 400 = 10x - \frac{1}{2}x^2 + 2x^2 - 40x + 400 = 1\frac{1}{2}x^2 - 30x + 400$.

D15b $O = 1\frac{1}{2}x^2 - 30x + 400 \Rightarrow \frac{dO}{dx} = O' = 3x - 30$.
$\frac{dO}{dx} = 0 \Rightarrow 3x = 30 \Rightarrow x = 10$. De oppervlakte is minimaal (er is slechts 1 kandidaat) voor $x = 10$.

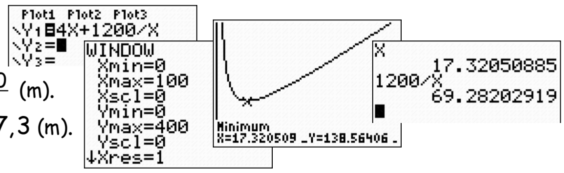
D16a $I = x \cdot \frac{1}{3}x \cdot h = 3\,000$ (cm³) $\Rightarrow x^2h = 9\,000$ (cm³) $\Rightarrow h = \frac{9000}{x^2}$ (cm).
$O = x \cdot \frac{1}{3}x + 2 \cdot x \cdot h + 2 \cdot \frac{1}{3}x \cdot h = \frac{1}{3}x^2 + 2\frac{2}{3}xh = \frac{1}{3}x^2 + \frac{8}{3}x \cdot \frac{9000}{x^2} = \frac{1}{3}x^2 + \frac{24000}{x}$ (cm²).



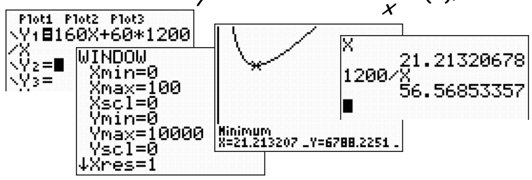

D16b $O = \frac{1}{3}x^2 + \frac{24000}{x} = \frac{1}{3}x^2 + 24\,000x^{-1} \Rightarrow \frac{dO}{dx} = O' = \frac{2}{3}x - 24\,000x^{-2} = \frac{2x}{3} - \frac{24000}{x^2}$
$\frac{dO}{dx} = 0 \Rightarrow \frac{2x}{3} = \frac{24000}{x^2} \Rightarrow 2x^3 = 3 \cdot 24\,000 \Rightarrow x^3 = 3 \cdot 12\,000 \Rightarrow x = \sqrt[3]{36\,000} \approx 33{,}0$ (cm).
De oppervlakte is minimaal (er is slechts 1 kandidaat) bij de afmetingen van 33,0 bij 11,0 bij 8,3 cm.
(de optie minimum is hier ook geoorloofd en geeft met van een een geschikt venster dezelfde $x$-waarde)




D17a $O = x \cdot y = 1200$ (m²) $\Rightarrow y = \frac{1200}{x}$.
De totale lengte van de afrastering is $L = 4x + y = 4x + \frac{1200}{x}$ (m).
$L$ is minimaal (de optie minimum mag toegepast worden) voor $x \approx 17{,}3$ (m).
De afmetingen van het stuk land zijn 17,3 bij 69,3 m.




D17b De kosten voor de afrastering zijn $K = 60(2x + y) + 20 \cdot 2x = 160x + 60y = 160x + 60 \cdot \frac{1200}{x}$ (€).
$K$ is minimaal (optie minimum mag) voor $x \approx 21{,}2$ (m).
De afmetingen van het stuk land zijn 21,2 bij 56,6 m.



**Gemengde opgaven 12. Differentiaalrekening**

G31a $l = \sqrt{(x_P)^2 + (y_P)^2} = \sqrt{p^2 + \left(4 - p^2\right)^2} = \sqrt{p^2 + \left(4 - p^2\right)\left(4 - p^2\right)} = \sqrt{p^2 + 16 - 4p^2 - 4p^2 + p^4} = \sqrt{p^4 - 7p^2 + 16}.$

G31b $l = \sqrt{p^4 - 7p^2 + 16} \Rightarrow \frac{dl}{dp} = \frac{1}{2 \cdot \sqrt{p^4 - 7p^2 + 16}} \cdot (4p^3 - 14p) = \frac{2p^3 - 7p}{\sqrt{p^4 - 7p^2 + 16}}.$

$\frac{dl}{dp} = 0 \Rightarrow \frac{2p^3 - 7p}{\sqrt{p^4 - 7p^2 + 16}} = \frac{0}{1}$ (teller = 0) $\Rightarrow 2p^3 - 7p = 0 \Rightarrow 2p(p^2 - 3\frac{1}{2}) = 0 \Rightarrow 2p = 0 \vee p^2 = 3\frac{1}{2}$ (met $p > 0$) $\Rightarrow p = \sqrt{3\frac{1}{2}}.$

Dus $l$ is minimaal (er is slechts 1 kandidaat) voor $p = \sqrt{3\frac{1}{2}}.$

G31c Deze cirkel heeft als straal de minimale lengte $l$ uit G31b (omdat de cirkel de parabool raakt).

$r = \sqrt{\left(\sqrt{3\frac{1}{2}}\right)^4 - 7\left(\sqrt{3\frac{1}{2}}\right)^2 + 16} = \sqrt{\left(3\frac{1}{2}\right)^2 - 7 \cdot 3\frac{1}{2} + 16} = \sqrt{12\frac{1}{4} - 24\frac{1}{2} + 16} = \sqrt{28\frac{1}{4} - 24\frac{1}{2}} = \sqrt{3\frac{3}{4}}.$

```
3.5²
                12.25
7*3.5
                 24.5
28.25-24.5
                 3.75
```

Dus de oppervlakte van deze cirkel is $O = \pi r^2 = \pi \cdot 3\frac{3}{4} = 3\frac{3}{4}\pi.$

G32a $\sin(x) = -\frac{1}{2}\sqrt{2}$

```
sin⁻¹(-1/2√(2))
        -.7853981634
Ans/π
                 -.25
```

$x = -\frac{1}{4}\pi + k \cdot 2\pi \vee x = 1\frac{1}{4}\pi + k \cdot 2\pi.$

G32b $\sin(2x - \frac{1}{4}\pi) \cdot \cos(x + \frac{1}{3}\pi) = 0$

```
sin⁻¹((0)
                    0
cos⁻¹((0)
         1.570796327
Ans/π
                   .5
```

$\sin(2x - \frac{1}{4}\pi) = 0 \vee \cos(x + \frac{1}{3}\pi) = 0$

$2x - \frac{1}{4}\pi = k \cdot \pi \vee x + \frac{1}{3}\pi = \frac{1}{2}\pi + k \cdot \pi$

$2x = \frac{1}{4}\pi + k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot \pi$

$x = \frac{1}{8}\pi + k \cdot \frac{1}{2}\pi \vee x = \frac{1}{6}\pi + k \cdot \pi.$

G32c $3\sin(2x - \frac{1}{3}\pi) = -1\frac{1}{2}\sqrt{3}$

```
sin⁻¹((-1/2√(3))
        -1.047197551
Ans/π
        -.3333333333
Ans▸Frac
                 -1/3
```

$\sin(2x - \frac{1}{3}\pi) = -\frac{1}{2}\sqrt{3}$

$2x - \frac{1}{3}\pi = -\frac{1}{3}\pi + k \cdot 2\pi \vee 2x - \frac{1}{3}\pi = \frac{4}{3}\pi + k \cdot 2\pi$

$2x = k \cdot 2\pi \vee 2x = \frac{5}{3}\pi + k \cdot 2\pi$

$x = k \cdot \pi \vee x = \frac{5}{6}\pi + k \cdot \pi.$

G33a $f(x) = x^2 \cdot \cos(2x) \Rightarrow f'(x) = 2x \cdot \cos(2x) + x^2 \cdot -\sin(2x) \cdot 2 = 2x\cos(2x) - 2x^2\sin(2x).$

G33b $g(x) = \sqrt{\sin(x) + x} \Rightarrow g'(x) = \frac{1}{2 \cdot \sqrt{\sin(x) + x}} \cdot (\cos(x) + 1) = \frac{\cos(x) + 1}{2 \cdot \sqrt{\sin(x) + x}}.$ Gebruik: $[\sqrt{x}]' = \frac{1}{2 \cdot \sqrt{x}}.$

G33c $h(x) = \left(\cos(x)\right)^2 - 2\sin(3x) \Rightarrow h'(x) = 2\cos(x) \cdot -\sin(x) - 2\cos(3x) \cdot 3 = -2\sin(x)\cos(x) - 6\cos(3x).$

G34a $f(x) = \sin^2(x) - \sin(x) = 0$

$\sin(x) \cdot \left(\sin(x) - 1\right) = 0$

$\sin(x) = 0 \vee \sin(x) = 1$

$x = k \cdot \pi \vee x = \frac{1}{2}\pi + k \cdot 2\pi.$ $x$ op $[0, 2\pi]$ geeft $x = 0 \vee x = \frac{1}{2}\pi \vee x = \pi \vee x = 2\pi.$

G34b $f(x) = \sin^2(x) - \sin(x) = \left(\sin(x)\right)^2 - \sin(x) \Rightarrow f'(x) = 2\sin(x) \cdot \cos(x) - \cos(x).$

$f'(x) = 0 \Rightarrow 2\sin(x) \cdot \cos(x) - \cos(x) = 0$

$\cos(x)\left(2\sin(x) - 1\right) = 0$

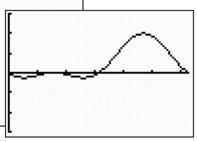
$\cos(x) = 0 \vee 2\sin(x) = 1$

$x = \frac{1}{2}\pi + k \cdot \pi \vee \sin(x) = \frac{1}{2}$

$x = \frac{1}{2}\pi + k \cdot \pi \vee x = \frac{1}{6}\pi + k \cdot 2 \vee x = \frac{5}{6}\pi + k \cdot 2\pi.$

```
Plot1 Plot2 Plot3
\Y1■sin(X)²-sin(X)
\Y2=
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=2π
 Xscl=1
 Ymin=-3
 Ymax=3
 Yscl=1
↓Xres=1
```



min. (zie plot) $f(\frac{1}{6}\pi) = -\frac{1}{4}$

max. $f(\frac{1}{2}\pi) = 2$

min. $f(\frac{5}{6}\pi) = -\frac{1}{4}$

max. $f(1\frac{1}{2}\pi) = 2.$

```
Y1(1/6π)
                 -.25
Y1(1/2π)
                    0
Y1(5/6π)
                 -.25
```

```
Y1(3/2π)
                    2
```

$f'(x) = 0$ (met $0 \le x \le 2\pi$) $\Rightarrow x = \frac{1}{6}\pi \vee x = \frac{1}{2}\pi \vee x = \frac{5}{6}\pi \vee x = 1\frac{1}{2}\pi.$ (de extreme waarden hierboven)

G34c $y_A = f(\pi) = \sin^2(\pi) - \sin(\pi) = 0$ en $rc_{raaklijn} = f'(\pi) = 2\sin(\pi) \cdot \cos(\pi) - \cos(\pi) = 0 - -1 = 1.$

$k$: $y = x + b$ door $A(\pi, 0) \Rightarrow 1 \cdot \pi + b = 0 \Rightarrow b = -\pi.$ Dus $k$: $y = x - \pi.$

G35a $I = x \cdot 2x \cdot h = 12$ (m³) $\Rightarrow h = \frac{12}{2x^2} = \frac{6}{x^2}$ (m).

$K = 120 \cdot x \cdot 2x + 80 \cdot (2 \cdot x \cdot h + 2x \cdot h) = 240x^2 + 80 \cdot 4xh = 240x^2 + 320x \cdot \frac{6}{x^2} = 240x^2 + \frac{1920}{x}$ (€).

```
320*6
                 1920
```

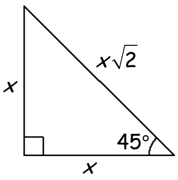
G35b $K = 240x^2 + \frac{1920}{x} = 240x^2 + 1920x^{-1} \Rightarrow \frac{dK}{dx} = K' = 480x - 1920x^{-2} = 480x - \frac{1920}{x^2}.$

$\frac{dK}{dx} = 0 \Rightarrow \frac{480x}{1} = \frac{1920}{x^2} \Rightarrow 480x^3 = 1920 \Rightarrow x^3 = \frac{1920}{480} = 4 \Rightarrow x = \sqrt[3]{4} \approx 1{,}59$ (m).

```
1920/480
                    4
```

```
3×√4→X
         1.587401052
2X
         3.174802104
6/X²
         2.381101578
```

De kosten zijn minimaal (er is slechts 1 kandidaat) bij de afmetingen van 1,59 bij 3,17 bij 2,38 m.



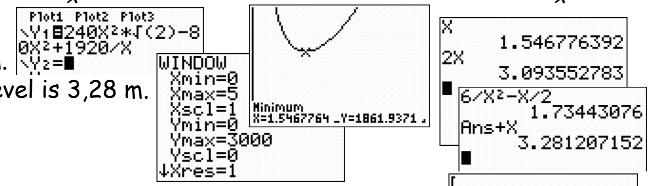

G35c $O_{\text{zijaanzicht}} = x \cdot h + \frac{1}{2} x \cdot x \Rightarrow I = (xh + \frac{1}{2}x^2) \cdot 2x = 2x^2h + x^3$.

G35d $I = 12$ (m³) $\Rightarrow 2x^2h = 12 - x^3 \Rightarrow h = \frac{12 - x^3}{2x^2} = \frac{12}{2x^2} - \frac{x^3}{2x^2} = \frac{6}{x^2} - \frac{x}{2}$ (m).

$K = 120 \cdot 2x \cdot x \cdot \sqrt{2} + 80 \cdot 2x \cdot h + 80 \cdot (xh + \frac{1}{2}x^2) \cdot 2 = 240x^2 \cdot \sqrt{2} + 160xh + 160xh + 80x^2$

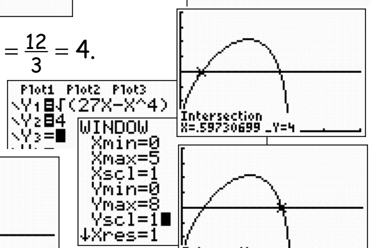
$= 240x^2 \cdot \sqrt{2} + 320x \cdot (\frac{6}{x^2} - \frac{x}{2}) + 80x^2 = 240x^2 \cdot \sqrt{2} + \frac{1920}{x} - 160x^2 + 80x^2 = 240x^2 \cdot \sqrt{2} - 80x^2 + \frac{1920}{x}$ (€).

G35e $K$ is minimaal (optie minimum geoorloofd) voor $x \approx 1{,}55$ (m).
De afmetingen van het vloeroppervlak zijn 1,55 bij 3,09 m.
De hoogte aan de voorkant is  m en de hoogte tegen de gevel is 3,28 m.

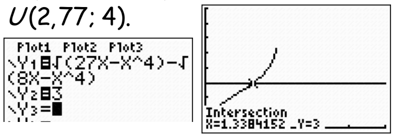



G36a $f(x) = 0 \Rightarrow \sqrt{27x - x^4} = 0$
$27x - x^4 = 0$
$x(27 - x^3) = 0$
$x = 0 \vee 27 = x^3$
$x = 0 \vee x = \sqrt[3]{27} = 3$.
Dus $OS = 3$.

$O_{\Delta OST} = 6$.
Dus $\frac{1}{2} \cdot OS \cdot y_T = \frac{1}{2} \cdot 3 \cdot y_T = 1\frac{1}{2} \cdot y_T = 6 \Rightarrow y_T = \frac{6}{1\frac{1}{2}} = \frac{12}{3} = 4$.
$y = \sqrt{27x - x^4} = 4$ (niet algebraïsch op te lossen ⇒ )
intersect geeft $x \approx 0{,}60 \vee x \approx 2{,}77$.
Dus $T(0{,}60;\ 4)$ en $U(2{,}77;\ 4)$.

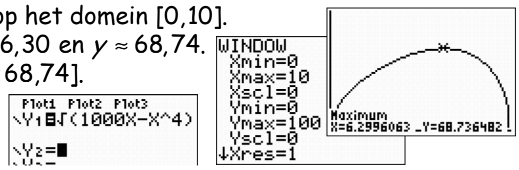



G36b $AB = f(p) - g(p) = 3 \Rightarrow \sqrt{27x - x^4} - \sqrt{8x - x^4} = 3$.
Intersect geeft $p \approx 1{,}34$.




G36c $h(x) = \sqrt{cx - x^4}$ met domein [0,10].
Dus $h(0) = 0$ en $h(10) = 0$.
$h(10) = \sqrt{10c - 10\,000} = 0$
$10c - 10\,000 = 0$
$10c = 10\,000$
$c = 1000$.

Dus $h(x) = \sqrt{1000x - x^4}$ op het domein [0,10].
Optie maximum geeft $x \approx 6{,}30$ en $y \approx 68{,}74$.
Dus het bereik van $h$ is [0;68,74].




G36d $h(x) = \sqrt{cx - x^4} \Rightarrow h'(x) = \frac{1}{2 \cdot \sqrt{cx - x^4}} \cdot (c - 4x^3) = \frac{c - 4x^3}{2 \cdot \sqrt{cx - x^4}}$.

$h$ heeft een maximum voor $x = 1{,}5 \Rightarrow h'(1{,}5) = 0 \Rightarrow \frac{c - 4 \cdot 1{,}5^3}{2 \cdot \sqrt{1{,}5c - 1{,}5^4}} = \frac{0}{1} \Rightarrow c - 4 \cdot 1{,}5^3 = 0 \cdot \sqrt{...} \Rightarrow c = 4 \cdot 1{,}5^3 = 13{,}5$.

```
4*1.5^3
                13.5
```

G37a $f(3) = f(-3) = 3^4 - 16 = 81 - 16 = 65$.
Dus de grafiek van $f$ is 65 omlaag verschoven.

G37b $f(x) = x^4 - 16 \Rightarrow f'(x) = 4x^3$
$\text{rc}_m = f'(2) = 4 \cdot 2^3 = 4 \cdot 8 = 32$
$m$: $y = 32x + b$ door $(-2, 0) \Rightarrow 32 \cdot -2 + b = 0 \Rightarrow b = 64$. Dus $m$: $y = 32x + 64$.

G37c $g(x) = x^3(x^4 - 16)$ (productregel of) $= x^7 - 16x^3 \Rightarrow g'(x) = 7x^6 - 48x^2$.
$g'(x) = 7x^6 - 48x^2 = 0 \Rightarrow 7x^2(x^4 - \frac{48}{7}) \Rightarrow x^2 = 0 \vee x^4 = \frac{48}{7} \Rightarrow x = 0 \vee x = \pm\sqrt[4]{\frac{48}{7}}$.
De $x$-coördinaten van de toppen zijn $-\sqrt[4]{\frac{48}{7}}$ en $\sqrt[4]{\frac{48}{7}}$ (bij $x = 0$ geen top maar een buigpunt).

G38a $A_{\text{balk}} = 2 \cdot 7{,}5 \cdot 4 + 2 \cdot 7{,}5 \cdot 10 + 2 \cdot 4 \cdot 10 = 60 + 150 + 80 = 290$ (cm²).
$A_{\text{cilinder}} = 2 \cdot \pi \cdot 3^2 + 2\pi \cdot 3 \cdot 10{,}6 \approx 256$ (cm²).
Omdat $V_{\text{balk}} = V_{\text{cilinder}}$ hoort de kleinste $F$ bij de verpakking met de kleinste oppervlakte.
Dus de cilindervormige verpakking heeft de kleinste $F$-waarde.

```
2*7.5*4+2*7.5*10
+2*4*10
                 290
2π*3²+2π*3*10.6
         256.3539605
```

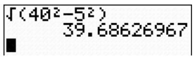
G38b $20 < h < 40 \Rightarrow 20 < \frac{8000}{\pi r^2} < 40$
$20 < \frac{8000}{\pi r^2}$ én $\frac{8000}{\pi r^2} < 40$
$20\pi r^2 < 8\,000$ én $40\pi r^2 > 8\,000$
$r^2 < \frac{8000}{20\pi}$ én $r^2 > \frac{8000}{40\pi}$
$(0 <) r < \sqrt{\frac{8000}{20\pi}} \approx 11{,}3$ én $r > \sqrt{\frac{8000}{40\pi}} \approx 8{,}0$.
Dus $8{,}0 < r < 11{,}3$

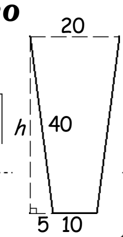
```
√(8000/(20π))
         11.28379167
√(8000/(40π))
         7.978845608
```

G38c $F = \frac{2}{r} + \frac{\pi r^2}{4000} = 2r^{-1} + \frac{\pi}{4000}r^2$ geeft
$\frac{\text{d}F}{\text{d}r} = F' = -2r^{-2} + \frac{\pi}{4000} \cdot 2r = -\frac{2}{r^2} + \frac{\pi r}{2000}$.
$\frac{\text{d}F}{\text{d}r} = -\frac{2}{r^2} + \frac{\pi r}{2000} = 0$
$\frac{\pi r}{2000} = \frac{2}{r^2}$
$\pi r^3 = 4\,000$
$r^3 = \frac{4000}{\pi} \Rightarrow r = \sqrt[3]{\frac{4000}{\pi}} \approx 10{,}8$ (cm).

```
3ˣ√(4000/π)
          10.8385214
```

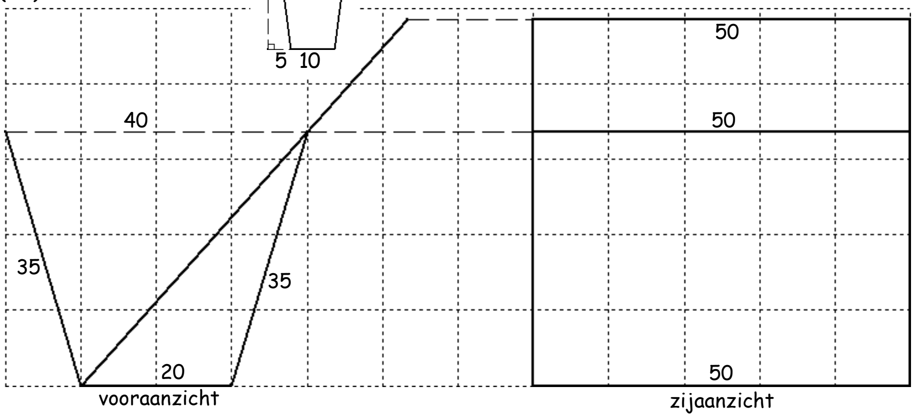
G39a Bestudeer het vooraanzicht hiernaast.
$90 - 10 = 80$ en $80 : 2 = 40$ (cm).
Pythagoras: $h = \sqrt{40^2 - 5^2} \approx 39{,}7$ (cm).

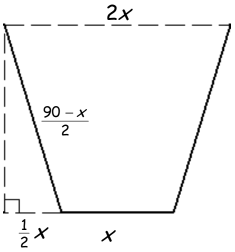



G39b Zie de hiernaast op schaal 1 : 10
(het vooraanzicht en) zijaanzicht.

G39c Maak een schets met de gegevens. Zie hieronder.



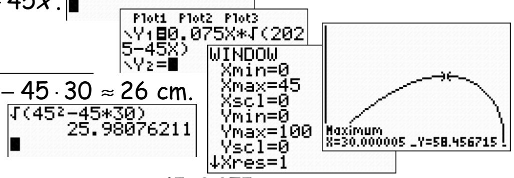




Pythagoras:

$$h = \sqrt{\left(\frac{90-x}{2}\right)^2 - \left(\frac{1}{2}x\right)^2} = \sqrt{\left(45 - \frac{1}{2}x\right)^2 - \frac{1}{4}x^2} = \sqrt{\left(45 - \frac{1}{2}x\right)\left(45 - \frac{1}{2}x\right) - \frac{1}{4}x^2}$$

$$= \sqrt{2025 - 45 \cdot \frac{1}{2}x - \frac{1}{2}x \cdot 45 + \frac{1}{4}x^2 - \frac{1}{4}x^2} = \sqrt{2025 - 45x}.$$


G39d $I = 0{,}075x \cdot \sqrt{2025 - 45x}$ optie maximum (is toegestaan) $\Rightarrow x = 30$ (cm).
De inhoud is maximaal (er is slechts 1 kandidaat) bij $x = 30$ en $h = \sqrt{2025 - 45 \cdot 30} \approx 26$ cm.




Of (algebraisch met de afgeleide)

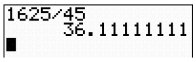
$I = 0{,}075x \cdot \sqrt{2025 - 45x}$

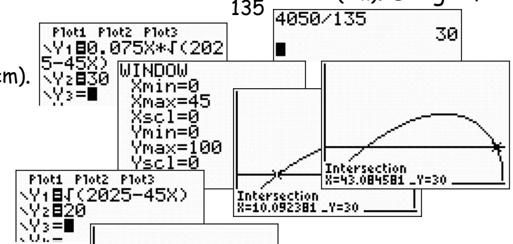
$$\frac{dI}{dx} = 0{,}075 \cdot \sqrt{2025-45x} + 0{,}075x \cdot \frac{1}{2 \cdot \sqrt{2025-45x}} \cdot -45 = 0{,}075 \cdot \sqrt{2025-45x} + \frac{-45 \cdot 0{,}075x}{2 \cdot \sqrt{2025-45x}}.$$

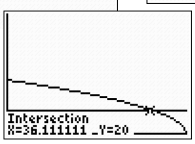
$$\frac{dI}{dx} = 0 \Rightarrow \frac{0{,}075 \cdot \sqrt{2025-45x}}{1} = \frac{45 \cdot 0{,}075x}{2 \cdot \sqrt{2025-45x}} \Rightarrow 2 \cdot \cancel{0{,}075} \cdot (2025 - 45x) = 45 \cdot \cancel{0{,}075}x$$

$4050 - 90x = 45x \Rightarrow 4050 = 135x \Rightarrow x = \frac{4050}{135} = 30$ (cm). Dit geeft dan (zoals hierboven) $h \approx 26$ (cm).

39e $I = 0{,}075x \cdot \sqrt{2025 - 45x} \geq 30$ (dm$^3$).
$I = 30$ intersect $\Rightarrow x \approx 10{,}1 \vee x \approx 43{,}1$ (cm).
$I \geq 30$ (zie een plot) $\Rightarrow 10{,}1 \leq x \leq 43{,}1$.
$h = \sqrt{2025 - 45x} \geq 20$ (cm) intersect of
$2025 - 45x \geq 400$
$-45x \geq -1625$
$x \leq \frac{1625}{45} \approx 36{,}1$ (cm).




$h \geq 20$ én $I \geq 30 \Rightarrow x \leq 36{,}1$ én $10{,}1 \leq x \leq 43{,}1$.
Dus $10{,}1 \leq x \leq 36{,}1$ (cm).

## Voorkennis 3 Differentiëren (bladzijde 156)

7a $f(x)=2x^3+3x^2 \Rightarrow f'(x)=6x^2+6x.$

7b $g(x)=\frac{1}{2}x^4-x^3+1 \Rightarrow g'(x)=2x^3-3x^2.$

7c $h(x)=x^2(x^2-2)=x^4-2x^2 \Rightarrow h'(x)=4x^3-4x.$

7d $k(x)=(4x^2+1)^2=(4x^2+1)(4x^2+1)=16x^4+4x^2+4x^2+1=16x^4+8x^2+1 \Rightarrow k'(x)=64x^3+16x.$

8a $f(p)=4p^3-\frac{1}{2}p \Rightarrow f'(p)=12p^2-\frac{1}{2}.$

8b $g(q)=\frac{1}{5}q^2+2q+1 \Rightarrow g'(q)=\frac{2}{5}q+2.$

8c $s(t)=5t^2+\frac{t+1}{2}=5t^2+\frac{1}{2}t+\frac{1}{2} \Rightarrow s'(t)=10t+\frac{1}{2}.$

8d $N(t)=0{,}01t^3-0{,}05t^2+0{,}1t \Rightarrow N'(t)=0{,}03t^2-0{,}1t+0{,}1.$

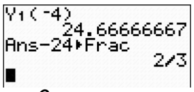
## Voorkennis 4 Extremen berekenen (bladzijde 157)

9a $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-12x-10 \Rightarrow f'(x)=x^2+x-12.$

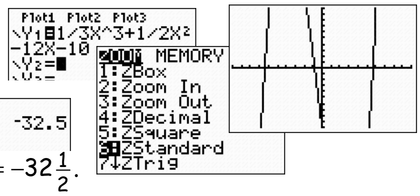
$f'(x)=0 \Rightarrow x^2+x-12=0$

$(x+4)(x-3)=0$

$x=-4 \vee x=3.$







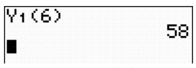

Maximum (zie een plot) $f(-4)=24\frac{2}{3}$ en minimum (zie een plot) $f(3)=-32\frac{1}{2}.$

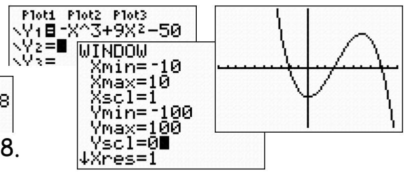
9b $g(x)=-x^3+9x^2-50 \Rightarrow g'(x)=-3x^2+18x.$

$g'(x)=0 \Rightarrow -3x^2+18x=0$

$-3x(x-6)=0$

$x=0 \vee x=6.$


Minimum (zie een plot) $g(0)=-50$ en maximum (zie een plot) $g(6)=58.$

## Voorkennis 5 Machten met negatieve en/of gebroken exponenten

(bladzijden 158 en 159)

10a $x^{-5}=\frac{1}{x^5}.$

10b $x^{\frac{1}{3}}=\sqrt[3]{x^1}=\sqrt[3]{x}.$

10c $x^{3\frac{1}{2}}=x^3\cdot x^{\frac{1}{2}}=x^3\cdot\sqrt{x}.$

10d $x^{-2\frac{1}{2}}=\frac{1}{x^{2\frac{1}{2}}}=\frac{1}{x^2\cdot x^{\frac{1}{2}}}=\frac{1}{x^2\cdot\sqrt{x}}.$

10e $6x^{\frac{1}{3}}=6\cdot\sqrt[3]{x^1}=6\cdot\sqrt[3]{x}.$

10f $5x^{-\frac{1}{4}}=5\cdot\frac{1}{x^{\frac{1}{4}}}=\frac{5}{\sqrt[4]{x}}.$

10g $\left(x^{\frac{1}{3}}\right)^2=x^{\frac{2}{3}}=\sqrt[3]{x^2}.$

10h $\left(\frac{1}{x}\right)^{-5}=\left(x^{-1}\right)^{-5}=x^5.$

11a $x^4\cdot\sqrt{x}=x^4\cdot x^{\frac{1}{2}}=x^{4\frac{1}{2}}.$

11b $\frac{x^{-2}}{x^3}=x^{-2-3}=x^{-5}.$

11c $\frac{x}{\sqrt[4]{x}}=\frac{x^1}{x^{\frac{1}{4}}}=x^{1-\frac{1}{4}}=x^{\frac{3}{4}}.$

11d $\frac{\sqrt[3]{x}}{x^2}=\frac{x^{\frac{1}{3}}}{x^2}=x^{\frac{1}{3}-2}=x^{-1\frac{2}{3}}.$

11e $\frac{x\cdot\sqrt{x}}{x^2}=\frac{x^1\cdot x^{\frac{1}{2}}}{x^2}=\frac{x^{1\frac{1}{2}}}{x^2}=x^{1\frac{1}{2}-2}=x^{-\frac{1}{2}}.$

11f $\frac{x^4}{x\cdot\sqrt{x}}=\frac{x^4}{x^1\cdot x^{\frac{1}{2}}}=\frac{x^4}{x^{1\frac{1}{2}}}=x^{4-1\frac{1}{2}}=x^{2\frac{1}{2}}.$

11g $\left(x\cdot\sqrt{x}\right)^2=\left(x^1\cdot x^{\frac{1}{2}}\right)^2=\left(x^{1\frac{1}{2}}\right)^2=x^3.$

11h $\left(x\cdot\sqrt[3]{x}\right)^2=\left(x^1\cdot x^{\frac{1}{3}}\right)^2=\left(x^{1\frac{1}{3}}\right)^2=x^{2\frac{2}{3}}.$